The evaluation of liquid mixing effects on a sieve plate using unsteady
and steady state tracer techniques

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Abstract—A rectangular sieve plate 5 ft 8½ in. long, 13½ in. wide with ½ in. diameter holes on 2-in. triangular centres has been used to evaluate liquid mixing effects in sieve-tray columns. The degree of mixing has been measured in terms of a longitudinal eddy coefficient \( D_L \) by an unsteady-state tracer technique.

Using the air-water system a series of liquid mixing tests have been made with liquid hold-ups varying between 2-3½ in. and air rates 1-4 ft/sec. With 2-in. and 1-in. liquid hold-ups the \( D_L \) values were considered to be related to vertical oscillations in the liquid that developed at air velocities of about 3-0 ft/sec, the indication being that the oscillations reduced the degree of liquid mixing. The \( D_L \) values obtained for liquid hold-ups of 2 to 3½ in. have been correlated in terms of the liquid hold-up \( (h_L, ft) \) and the superficial air velocity \( (U, ft/sec) \) as follows:

\[
D_L = 0.01298U^{1.44} + 0.3024h_L - 0.0065
\]

Longitudinal eddy coefficients \( (D_L) \) have also been measured for the same sieve plate but using liquid cross-flowing conditions by a steady-state tracer technique. Using the air-water system, values of \( D_L \) varied between 0.0135 and 0.25 ft²/sec over the wide range of operating conditions investigated. No explanation was apparent for deviations from the eddy-diffusion model which occurred when the froth velocity on the plate fell below 0.23 ft/sec. At air rates of about 7-0 ft/sec pulsations on the plate occurred corresponding to a rapid increase in the values of \( D_L \). All the \( D_L \) values obtained on the plate using liquid cross-flowing conditions have been successfully correlated in terms of the liquid hold-up and the froth momentum on the plate.

INTRODUCTION

The presence of a concentration gradient in the liquid on bubble plates was realized as early as 1936 but until recently no methods were available for predicting its nature and effect upon the plate efficiencies of distillation and absorption columns. The concentration gradient is a function of the amount of back-mixing of the liquid that occurs on a plate. If the liquid on a plate is completely mixed, the point or local efficiency is the same as the plate efficiency. In practice, and particularly in commercial-size columns, only partial mixing of the liquid takes place with the result that plate efficiencies may be greater than 100 per cent. Methods of calculating the degree of mixing for these situations are obviously desirable if plate efficiencies are to be calculated with any confidence.

Several concepts have been proposed to describe the liquid mixing process, the first attempt being the pool concept of KIRSCHAUM [1, 2]. He assumed that a plate could be divided in the direction of liquid flow into several equal-sized, perfectly mixed pools, and that liquid flowed from one pool into the next until it reached the outlet weir. A plate with a single pool corresponded to a perfectly mixed plate and one with an infinite number of pools to an unmixed plate. Recently GAUTREAU and O'CONNELL [3] have revived the pool concept of KIRSCHAUM, presenting an equation relating point and plate efficiencies.

Other concepts include the recycle stream concept of OLIVER and WATSON [4] and WARZEL [5] who assumed that liquid mixing is effected by a certain fraction of the liquid at the outlet weir being recirculated to the inlet weir, where it is mixed with the incoming liquid. The parameter which characterizes the mixing is calculated from the difference in the experimentally determined liquid concentrations preceding and immediately following the inlet weir.

JOHNSON and MARANGOZIS [6] have carried out some mixing studies on a perforated plate in which
the splashing of the liquid was considered to be the major mixing process. They established a relationship between point and plate efficiency in terms of a mixing factor due to liquid splashing.

The degree of mixing on a bubble plate has also been characterized by measurement of the residence times of the liquid elements flowing across the plate. Point and plate efficiencies have been related in terms of the residence time distribution function by Foss et al. [7]. The main advantage of this method is that it can be used even when by-passing, and channelling of the liquid occur. Its disadvantage is the complexity and tediousness of the analytical procedure.

Finally, the eddy diffusion concept [8–10] supposes that liquid mixing on a plate can be interpreted by the simple law of diffusion. Danckwerts [11] was the first to point out that longitudinal mixing of fluids in continuous-flow systems could be treated like molecular diffusion.

The purpose of the work described here was to see if liquid mixing on a sieve plate could be interpreted by an eddy-diffusion mechanism when using an unsteady-state tracer technique. Also to extend the application of the eddy diffusion concept to the same rectangular sieve plate but using liquid cross-flowing conditions and to obtain liquid-mixing values suitable for the prediction of sieve-plate efficiencies.

**APPARATUS AND EXPERIMENTAL PROCEDURE**

The apparatus shown in Fig. 1 consisted of a rectangular aluminium column fitted with an aluminium sieve plate (A) 5 ft 8½ in. long, 13½ in. wide and 1½ in. thick. The sieve plate had 1 3 in. diameter holes spaced on 2 in. triangular centres giving a 5 per cent free area based on the total bubbling area. The column was designed so that the plate could be easily removed and different types of plate fitted if required. In the static tests described here no flow of liquid was to take place along the length of the plate, so high weirs were fitted at either end of the plate to prevent the liquid placed on it from being lost. Air to the plate was supplied by a fan (B) capable of delivering 2100 ft³/min against a head of 10 in. water gauge and was metered by a venturi (C) placed in the air inlet duct. The air entered the column through a three-way manifold underneath the tray, the air flow being further smoothed out by a layer of Spraypak, placed between the plate and the manifold. The air distribution across the plate was tested for uniformity by measuring the velocity through the holes, and a variation of ±2 per cent found across the plate. A metal trough with a capacity of 0.106 ft³ was fitted at one end of the plate about 6–10 in. above the plate surface, the height being dependent on the amount of liquid placed on the plate. A handle outside the column was fastened to the trough so that the dye to be used in the experiments could be introduced speedily and uniformly across the width of the plate by turning the handle.

The technique used for the measurement of the longitudinal eddy-diffusion coefficients on the static plate was to inject a small volume of dye uniformly across the width of one end of the plate and to measure the rate of increase of colour concentration at a known point at the other end of the plate. The procedure for a run was briefly as follows. With the air rate adjusted to the required value, a known
volume of water was placed on the plate, care being taken to see that none of the water wept through the holes in the plate. The correct volume of dye was placed in the trough (usually $\frac{1}{2}$ of the volume of water on the plate) and the dye speedily injected by turning the handle of the trough. Immediately before the dye was injected, a secondary weir was inserted across the plate about 3 in. from the end weir. The dye was able to mix completely with the water trapped between the two weirs before the secondary weir was removed and the dye commenced to spread across the plate by the normal mixing process. A stop-clock was started simultaneously with the raising of the secondary weir and 3-ml liquid samples were collected with a small glass hypodermic syringe at a predetermined sample point at the other end of the plate. Samples were taken every few seconds until the mixing process was complete. The correct rate of sampling was determined by a trial run to obtain an approximate idea of the concentration-time curve for the test. The dye used was Azo Geramine 2G, 200 a solution of 0-156 lb/ft$^3$ of water being used in the diffusivity tests. An absorptiometer was calibrated and used for determining the concentrations $(C)$ of the samples taken from the plate and the initial concentration of the dye $(C_0)$.

For the steady-state liquid-mixing experiments water from the mains supply was fed to a 30-gal galvanized tank (D) and then circulated to the inlet weir of the plate by a centrifugal pump (E) delivering 80 gall/min against a head of 14 ft of water. The water rate was measured by an orifice plate (F) in the water line. In the tests described here the water was not recirculated but passed to the drain after flowing across the plate. The plate was fitted with inlet and outlet weirs which could be adjusted in $\frac{1}{2}$-in. increments from 1 to 6 in. A concentrated solution of common salt which was to be used in the tests was stored in a 30-gal galvanized tank (G). The salt solution was pumped by the centrifugal pump (H), which had a capacity of 120 gal/hr against a head of 5 ft of water, to a glass injector grid (K) fixed over the width of the plate 13-25 in. upstream from the outlet weir. Holes $\frac{1}{4}$ in. in diameter were drilled at $\frac{1}{4}$-in. intervals along the injector grid. The grid was tested for uniformity of distribution by collecting and measuring the salt solution leaving equal lengths of the grid. The variation in liquid rate along the grid was found to be negligible. The salt solution was metered to the plate by the orifice plate (L). Liquid samples from the plate were obtained by means of sample points located in the floor of the plate and spaced 2 in. apart down the length of the plate. These sample points were connected by P.V.C. tubing through the column shell to the outside. Samples from the plate flowed by gravity, the rate of sampling being controlled by screw clips on the tubing. The salt concentration of each liquid sample was measured by a conductivity cell contained in a Mullard a.c. bridge circuit. Electrical conductivity varies appreciably with changes in temperature and for this reason it was necessary to analyse the samples at constant temperature.

The technique used for the measurement of the eddy diffusion coefficients $(D_L)$ on the flowing plate, was to continuously and uniformly inject a salt solution across the width of the plate near the outlet weir, and to measure the degree of tracer diffusion upstream. The theory of the method is given in the Appendix. The concentration of salt on the plate was limited to 0.0624 lb/ft$^3$, because above this concentration there was a non-linear relationship between the specific conductivity of the solution and concentration, resulting in less accurate analyses. The procedure for a run was briefly as follows. For a fixed outlet weir height, the air rate and water-flow rate were adjusted to the required values. Addition of salt solution was then made until samples taken from the plate showed that the salt concentration was less than 0.0624 lb/ft$^3$. All flow rates were held constant for 20 min while liquid samples were continuously withdrawn from the floor of the plate into 250-ml sample bottles; any excess liquid passed to the drain. The rate of sampling was adjusted by screw clips to about 250 ml every 3 min. A sample of mains water was also taken at the end of each run.

To calculate the $D_L$ values it was also necessary to know the froth velocity on the plate. This was calculated from the clear liquid hold-up on the plate and the liquid flow rate as follows

\[ \nu = \frac{Q_L}{h_L} \]  

(1)
where $Q_L =$ liquid rate, ft$^3$/sec per ft of weir length
$h_L =$ clear liquid hold-up on the plate, ft
$V =$ froth velocity, ft/sec

The clear liquid hold-ups ($h_L$) were measured by four sample taps mounted flush in the plate floor, and situated along the centre line of the plate. These were connected by P.V.C. tubing to external glass manometers. Variations in the manometer readings for a particular operating condition were found to be small (up to 5 per cent) and an average value was taken.

Froth height data for correlation purposes was also required over a wide range of operating conditions. The froth height in this work refers to the height above the plate of the boundary which separates the foaming liquid mass on the plate from the vapour and liquid droplets above it. This boundary was not clearly defined especially at higher air rates but it was found that the results could be reproduced to within ±0·5 in. Froth heights were measured by standing a graduated ruler vertically on the floor of the plate and gradually lowering a horizontal ruler until it just reached the top of the froth. Several readings were recorded at different parts of the plate and a mean value taken.

RESULTS AND DISCUSSION

Mixing values using the unsteady-state tracer technique

If $h$ represents the space initially occupied by the dye at zero time, $z_1$ the length of the plate, $t$ the distance to the sampling point and $C_0$ the initial concentration of the dye, then it can be shown, by the theoretical solution to the problem [10, 12], that the concentration $C$ at any point $l$ on the tray changes with time according to the following equation:

$$
C = \frac{1}{4} C_0 \left( \text{erfc} \frac{l - h}{2\sqrt{(D_E t)}} - \text{erfc} \frac{l + h}{2\sqrt{(D_E t)}} + \right.
$$

$$
+ \text{erfc} \frac{2z_1 - h - l}{2\sqrt{(D_E t)}} - \text{erfc} \frac{2z_1 + h - l}{2\sqrt{(D_E t)}} +
$$

$$
+ \text{erfc} \frac{2z_1 - h + l}{2\sqrt{(D_E t)}} - \text{erfc} \frac{2z_1 + h + l}{2\sqrt{(D_E t)}} + \ldots \right)
$$

(2)

where $D_E =$ the longitudinal eddy diffusion coefficient of mixing, ft$^2$/sec
$t =$ time, sec
$l, z_1, h =$ linear distances, ft
$h/z_1 =$ proportion by volume of dye to water on the plate

It was found for the range of conditions studied in this work that equation (2) converged rapidly requiring a maximum of six terms in any evaluation. The application of the equation is now illustrated by an example.

Sample calculation. $l = 5$ ft 4 in. $z_1 = 5$ ft 8 in. $z_1 = 24h$

Amount of water on the plate = 1·06 ft$^3$ (2 in. liquid hold-up)

Amount of dye injected 0·0443 ft$^3$

Air rate 2·01 ft/sec

Fig. 2. Theoretical increase of concentration at sample point.

Fig. 3. Experimental points with fitted theoretical curve.

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Table 1

<table>
<thead>
<tr>
<th>Time t (sec)</th>
<th>Absorptiometer Reading</th>
<th>Concentration C (lb/lb² × 10⁻⁴)</th>
<th>C/C₀</th>
<th>√(Dₑf)</th>
<th>Eddy-Diffusion Coefficient Dₑ (ft²/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>2.5</td>
<td>1.56</td>
<td>0.001</td>
<td>1.14</td>
<td>0.0324</td>
</tr>
<tr>
<td>55</td>
<td>8.5</td>
<td>5.8</td>
<td>0.0037</td>
<td>1.36</td>
<td>0.0339</td>
</tr>
<tr>
<td>70</td>
<td>15.3</td>
<td>10.8</td>
<td>0.0069</td>
<td>1.53</td>
<td>0.0334</td>
</tr>
<tr>
<td>85</td>
<td>22.7</td>
<td>16.4</td>
<td>0.0105</td>
<td>1.67</td>
<td>0.0329</td>
</tr>
<tr>
<td>100</td>
<td>29.1</td>
<td>21.2</td>
<td>0.0136</td>
<td>1.78</td>
<td>0.0322</td>
</tr>
<tr>
<td>115</td>
<td>35.5</td>
<td>25.9</td>
<td>0.0166</td>
<td>1.90</td>
<td>0.0314</td>
</tr>
<tr>
<td>130</td>
<td>40.5</td>
<td>30.2</td>
<td>0.0194</td>
<td>2.01</td>
<td>0.0313</td>
</tr>
<tr>
<td>150</td>
<td>48.6</td>
<td>37.0</td>
<td>0.0238</td>
<td>2.2</td>
<td>0.0326</td>
</tr>
<tr>
<td>170</td>
<td>54.2</td>
<td>41.8</td>
<td>0.0268</td>
<td>2.34</td>
<td>0.0323</td>
</tr>
<tr>
<td>190</td>
<td>58.9</td>
<td>45.8</td>
<td>0.0294</td>
<td>2.47</td>
<td>0.0324</td>
</tr>
<tr>
<td>210</td>
<td>62</td>
<td>48.7</td>
<td>0.0313</td>
<td>2.57</td>
<td>0.0317</td>
</tr>
<tr>
<td>230</td>
<td>66</td>
<td>52.6</td>
<td>0.0337</td>
<td>2.72</td>
<td>0.0323</td>
</tr>
</tbody>
</table>

The concentrations (C) at various times (t) obtained experimentally are shown in Table 1. From equation (2) a theoretical curve of C/C₀ vs. √(Dₑf) may be plotted for specific values of l, z₁, and h as shown in Fig. 2. This curve is for the case where z₁ = 24h. From the experimental results which were obtained for the same ratio of z₁ to h a plot of C/C₀ vs. √t may be drawn as shown in Fig. 3. Thus for each value of C/C₀, √t and (√Dₑf) are known and a value of Dₑ is calculable. The results for this test are shown in Table 1. A series of theoretical curves for fixed values of Dₑ are drawn on Fig. 3 and the theoretical curve which best fits the experimental points gives the value of Dₑ for the test, i.e. 0.0322 ft²/sec in the example given.

The first tests carried out were to see if the value of Dₑ varied with the sampling position on the plate. For equation (2) to hold true, Dₑ should be independent of the sampling position. At constant conditions of air rate and liquid hold-up, samples were taken from nine different positions on the plate and a maximum variation of 5-4 per cent between the Dₑ values was obtained.

The close agreement between the theoretical curves as predicted by equation (2) and the experimental concentration-time curves, gave support to the postulation that liquid mixing on a static sieve plate could be interpreted by an eddy-diffusion mechanism. Because of the small variation of Dₑ values with sampling position, a point 5 ft-1 in. from the dye-injection zone and on the centre line of the plate was selected as the sampling point for all future tests.

A series of liquid mixing tests were carried out with various liquid hold-ups on the plate, each hold-up being investigated for a range of air rates. Liquid hold-ups were varied between 3/4 in. and 3 1/2 in. and the air rates from 1 to 4 ft/sec. It was considered that this range of hold-ups covered the range of hydraulic conditions encountered in the normal operation of a sieve plate. Higher air rates could not be achieved because of the limitations of blower capacity. With liquid hold-ups on the plate between 2 in. and 3 1/2 in., Dₑ was found to be a function of both the air rate and the liquid hold-up on the plate (see Fig. 4). It was found that these results could be correlated by the following equation

\[ Dₑ = 0.01298U^{1.44} + 0.3024h_L - 0.0605 \]  

(3)

where Dₑ = the longitudinal eddy diffusion coefficient, ft²/sec

U = the superficial air velocity based on the bubbling area, ft/sec

h_L = the liquid hold-up on the plate, ft

A comparison of the experimental Dₑ values and those predicted by equation (3) are shown plotted in Fig. 5. The Dₑ values for a 1 1/4-in. liquid hold-up were found to be the same as for a 2-in. liquid hold-up.

Oscillation phenomenon. The Dₑ determinations for liquid hold-ups of 1 1/2 in. and 1 in. proved particularly interesting because of the inflexion in the Dₑ values.
with increasing air rate as shown in Fig. 6. This phenomenon was considered to be associated with vertical oscillations of the liquid in a direction perpendicular to the spread of the dye, which began to develop at an air velocity of approximately 3-3 ft/sec for a 1-in. hold-up. These oscillations became more pronounced as the air velocity was increased, and it was noticeable that they occurred over about one third of the plate length. This movement of liquid presumably tended to delay the diffusion of dye down the plate thus decreasing the rate of increase of dye concentration at the sampling point. Oscillations were also apparent in the 1-in. hold-up tests, the movement occurring initially at a slightly lower air rate than in the 1-in. hold-up tests. The oscillation phenomenon was not obtained with liquid hold-ups greater than 1 in. for the range of air rates investigated.

**Mixing values using the steady-state tracer technique**

The theoretical basis for obtaining liquid mixing data on bubble plates is given in the Appendix. The method of obtaining the mixing values is illustrated by the following example.

Water rate = 15 gal/min per ft of weir; Air rate to plate = 2·5 ft/sec.
Outlet weir height = 3 in.; Length of operating plate 55·25 in.
Clear liquid hold-up 1·94 in. Temperature 20°C.

**Table 2**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Specific conductivity at 25°C (mho)</th>
<th>Concentration (lb/ft² × 10⁻²)</th>
<th>(\frac{x - x_0}{x_0 - x_0})</th>
<th>l - w</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (at grid)</td>
<td>0·111 × 10⁻¹</td>
<td>492·0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3·14 × 10⁻⁴</td>
<td>121·0</td>
<td>0·246</td>
<td>0·036</td>
</tr>
<tr>
<td>3</td>
<td>1·06 × 10⁻⁴</td>
<td>28·4</td>
<td>0·0576</td>
<td>0·072</td>
</tr>
<tr>
<td>4</td>
<td>0·56 × 10⁻⁴</td>
<td>6·13</td>
<td>0·0125</td>
<td>0·108</td>
</tr>
<tr>
<td>5 (nearest inlet weir)</td>
<td>0·46 × 10⁻⁴</td>
<td>1·56</td>
<td>0·0032</td>
<td>0·145</td>
</tr>
<tr>
<td>6 (mains water)</td>
<td>0·425 × 10⁻⁴</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where:
- \(x\) = the salt concentration of the sample, lb/ft²
- \(x_0\) = the salt concentration of the mains water, lb/ft²
- \(x_w\) = the salt concentration at the injector grid, lb/ft²
- \(x\) = the distance from the inlet weir to the sampling point (ft)
- \(w\) = the total tray length (ft)

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![Graph showing eddy-diffusivity values in oscillating region of plate operation.](image)

**Fig. 6.** Eddy-diffusivity values in oscillating region of plate operation.

![Graph showing variation of salt concentration on plate.](image)

**Fig. 7.** Variation of salt concentration on plate.

Analysis of the samples taken at 2 in. intervals along the length of the plate, gave the results shown in Table 2.

A plot of \( \log[(x - x_0)/(x_e - x_0)] \) vs. \((l - w)\) is shown in Fig. 7. As indicated by equation (A.6) in the Appendix, such a plot is linear when \(N_{pe} \gg 1\), the slope of the line giving the value of the Peclet number \(N_{pe}\). For this case \(N_{pe} = 39.7\).

Now

\[
N_{pe} = \frac{Vz_1}{D_e}
\]  

(4)

where

\[
V = \frac{Q_L}{h_L}
\]

\[
\therefore V = \frac{15}{6.23 \times 60 \times 0.162} = 0.248 \text{ ft/sec}
\]

and

\[
D_e = \frac{Vz_1}{N_{pe}} = \frac{0.248 \times 55.25}{39.7 \times 12} = 0.0288 \text{ ft}^2/\text{sec}
\]

Several runs were carried out with a 1-in. outlet weir to determine the extent of mixing at low weir heights. It was found that with air velocities of up to 5-0 ft/sec back-mixing was only measurable at liquid rates below 15 gal/min ft of weir. For liquid rates above 15 gal/min ft and vapour rates below 4.5 ft/sec, it may be assumed that the liquid on the plate is approaching an unmixed state. However, all the results (see Table 3) that were obtained gave a linear plot of \( \log[(x - x_0)/(x_e - x_0)] \) vs. \((l - w)\).

Using a 2-in. outlet weir, \(D_e\) values were obtained for the following operating conditions: liquid rates 10-50 gal/min ft and air rates (1.5-4.8 ft/sec). These values are shown plotted in Fig. 8. Below a liquid rate of 10 gal/min ft, and at air flow rates above 2.5 ft/sec non-linear plots of \( \log[(x - x_0)/(x_e - x_0)] \) vs. \((l - w)\) were obtained.

With a 3-in. outlet weir, \(D_e\) values were obtained for the majority of operating conditions investigated and these are shown plotted in Fig. 9. Again however, at liquid rates below 15 gal/min ft, non-linear plots of \( \log[(x - x_0)/(x_e - x_0)] \) vs. \((l - w)\) were obtained. Deviations from the linear plot became more appreciable as the air rate increased.

The range of operating conditions within which the liquid mixing could be represented by a diffusional model was even more limited with the 4-in. outlet weir. \(D_e\) values could only be obtained for
liquid rates above 10 gal/min ft and air rates above 4 ft/sec. These results are given in Table 4.

It should be noted that although back-mixing was still appreciable at those operating conditions where non-linear plots were obtained, the Peclet number was still well above one so that the approximation invoked by equation (A.6) was still valid.

This work has shown that the assumed mixing model is not followed under all operating conditions. Fortunately, deviations occurred at operating conditions which have limited practical application. In this investigation, deviations occurred whenever the froth velocity across the plate fell below 0.23 ft/sec. When this occurred the amount of back-mixing was appreciable and any discrepancies in the assumed mixing model became more apparent. If limits do exist within which the model is applicable, it seems probable that outside these limits, certain essential conditions of the model are not being fulfilled.

In the application of the diffusion model to longitudinal mixing in continuous-flow systems, one of the basic conditions is that the lateral mixing of the tracer should be great enough to ensure a uniform tracer concentration at any given cross-section of the plate. Early work using a dye as the continuous tracer had shown that this condition was not completely satisfied. The dye was inclined to mix
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Table 4

<table>
<thead>
<tr>
<th>Liquid rate, L (gal/min ft weir length)</th>
<th>Air rate, U (ft/sec)</th>
<th>Weir height W (in.)</th>
<th>Peacock number N_{pe}</th>
<th>Eddy-diffusion coefficient D_E (ft^2/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>4.05</td>
<td>4</td>
<td>21.5</td>
<td>0.0624</td>
</tr>
<tr>
<td>25</td>
<td>4.16</td>
<td>4</td>
<td>26.14</td>
<td>0.0648</td>
</tr>
<tr>
<td>25</td>
<td>4.72</td>
<td>4</td>
<td>27.17</td>
<td>0.0668</td>
</tr>
<tr>
<td>35</td>
<td>4.08</td>
<td>4</td>
<td>29.93</td>
<td>0.0723</td>
</tr>
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<td>35</td>
<td>4.54</td>
<td>4</td>
<td>32.55</td>
<td>0.0704</td>
</tr>
<tr>
<td>50</td>
<td>4.03</td>
<td>4</td>
<td>33.2</td>
<td>0.085</td>
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<tr>
<td>50</td>
<td>4.55</td>
<td>4</td>
<td>35.59</td>
<td>0.0841</td>
</tr>
</tbody>
</table>

slightly more rapidly down one side of the plate than the other, the effect becoming more pronounced as the rate of mixing increased. It was thought that the above effect might be due to mal-distribution of the liquid or vapour but a thorough investigation proved this was not so. However when the plate was rotated through 180° about either axis, variations in the amount of back-mixing for the same operating conditions were quite noticeable. As the air and water distributions were unaffected by these changes, the indications were that these variations might have been caused by slight imperfections in the perforations. Although all the plate perforations were made on a drilling machine, it was impossible to ensure that every perforation was drilled at exactly 90° to the plate surface. It seems possible therefore that the degree of mixing may be affected to some extent by the accuracy with which the perforations are drilled or punched.

Pulsating region of plate operation

To obtain air velocities above 5-0 ft/sec the length of the sieve plate was reduced from 68.5 in. to 32.5 in. by sealing off the perforations with a waterproof adhesive tape. It was found that with a 1-in. outlet weir and a liquid rate of 10 gal/min ft, a pulsation of the froth on the plate commenced when the air rate was about 6.8 ft/sec. Increasing the air rate to 7.5 ft/sec increased the amplitude and violence of the pulsations. Increasing the liquid rate to 50 gal/min ft slightly damped the pulsations.

Using 2-, 3- and 4-in. outlet weirs, and a liquid rate of 10 gal/min ft, pulsations commenced when the air rate was about 7 ft/sec. Increasing the air rate up to 7.5 ft/sec produced the same effect as for the 1-in. outlet weir. Increasing the liquid rate up to 50 gal/min ft made the pulsations more pronounced and also slightly lowered the air rate at which they commenced.

From Fig. 10, it is interesting to note that the $D_E$ values increased rapidly with increasing air rate in the pulsating region of plate operation. This would indicate that the pulsations greatly increased back-mixing. These findings are contrary to the static plate tests, where in the oscillating region, $D_E$ was shown to decrease with air rate before increasing again. This discrepancy is considered to be due to differences in the nature of the froth movements on the two plates. On the static plate a uniform vertical oscillation was obtained between the plate sides, perpendicular to the flow of dye. However on the

![Fig. 10. Eddy-diffusivity values at high air rates.](image-url)
flowing plate, the pulsations tended to be of a sinusoidal nature running from one side of the plate to the other, along the length of the plate. The first type of motion mentioned would tend to suppress back-mixing, while the second type would increase it.

McAllister and Plank [13] reported a periodic pulsation of the frothing mass of liquid in a direction perpendicular to the liquid flow on small-diameter sieve- and bubble-cap plates. Using the air-water system, pulsations commenced at an air rate of 6 ft/sec. For a bubble-cap plate with a liquid path length of 11·5 in. they were able to show that the number of well-mixed pools in the Gautreaux and O’Connell theory [3] increased under pulsating operating conditions, thus indicating a decrease in the amount of back-mixing and resulting in a closer approach to an unmixed plate.

**Correlation of \( D_E \) results**

It was found in this work that the \( D_E \) values generally increased as the air rate, water rate and outlet weir height were increased. This trend was also typical of the results presented in Ref. [14]. Here the \( D_E \) values obtained on a bubble cap plate using 3-in. diameter caps were correlated by the following equation

\[
D_E^2 = 0.0124 + 0.017U + 0.0052L + 0.0150W
\]

where \( L = \text{liquid rate U.S. gal/min ft length of weir.} \) This equation was stated to be also suitable for predicting \( D_E \) values for sieve plates. A comparison of the results presented in this paper with those predicted by equation (5) is shown in Fig. 11. Also shown in this plot are the results of Gilbert [15]. From the scatter of the results it would appear that the parameters chosen in the equation are not the best available.

- Foss et al. [7] suggested a method of correlating liquid mixing results in terms of the liquid hold-up and the froth momentum on the plate. They used a plot of rate-of-exchange of variance per unit of liquid hold-up vs. froth momentum to correlate liquid residence-time data, but this can easily be converted for use with \( D_E \) measurements to give a plot of \( 2D_E/V^2h_L \) vs. \( h_L/V/h_f \). This relationship was found to correlate all the results obtained (see Fig. 12). Both lines on the graph fit the equation

\[
\frac{2D_E}{V^2h_L} = C_1 \left( \frac{h_LV}{h_f} \right)^{-C_2}
\]

where \( C_1 \) and \( C_2 \) are constants

\[
h_f = \text{froth height}, ft
\]

\[
h_L = \text{liquid hold-up}, ft
\]

\[
V = \text{froth velocity}, ft/sec
\]

Within the range of operating conditions investigated the results obtained on the plate with 2-, 3- and 4-in. outlet weirs were best represented by the equation (7).

\[
\frac{2D_E}{V^2h_L} = 0.0098 \left( \frac{h_LV}{h_f} \right)^{-2.91}
\]

The results obtained with a 1-in. outlet weir were correlated by equation (8).

\[
\frac{2D_E}{V^2h_L} = 0.0024 \left( \frac{h_LV}{h_f} \right)^{-3.02}
\]

Average \( D_E \) values calculated for those cases where the mixing results gave a non-linear plot when expressed by equation (A.6) were also found to be correlated by equation 7. These particular results are not shown plotted on Fig. 12. It was also found that all the mixing results obtained in the pulsating region of plate operation were correlated by either equations (7) or (8) depending on the height of weir used. The best lines through the liquid-mixing
The evaluation of liquid mixing effects on a sieve plate using unsteady and steady state tracer techniques.

results, which were obtained on a rectangular bubble-cap plate (31 in. x 9 in.) with those presented here indicates that the degree of mixing on a sieve plate is higher than on a bubble-cap plate under the same conditions of froth momentum. This confirms the earlier observations (see Fig. 11) that, under the same conditions of water rate, air rate and outlet weir height, the majority of sieve plate $D_e$ values are higher than the corresponding results on the bubble-cap plate. The deviations between the presented results and those of GILBERT are probably due to the different methods used to determine the liquid hold-up on the plate. GILBERT attempted to measure the actual volume of water on the plate, whilst in the work described here and in the A.I.Ch.E. programme, liquid hold-up values were obtained from manometers attached to the floor of the plate. The former method tends to give higher values than the manometer method for the same operating conditions. No obvious explanation is apparent for the greater deviation of the presented results from those of Foss et al., although it is to be noted that Foss et al. determined the residence time distribution of liquid on a sieve plate and this was then converted to a $D_e$ value for comparison purposes in Fig. 13.

LIQUID HOLD-UP AND FROTH-HEIGHT CORRELATIONS

* From the above considerations, it is apparent that $D_e$ values are best correlated in terms of the liquid hold-up, froth-height and froth velocity. It is preferable that these variables should themselves be related to the air rate, water rate and outlet weir height.

Liquid hold-up data for the sieve plate with 2-, 3- and 4-in. outlet weir heights were found to be well represented by the following equation:

$$h_L = 0.078 + 0.00145L + 0.031W - 0.013U$$  (9)

where $h_L =$ clear liquid hold-up, ft
$L =$ liquid rate, gal/min
$W =$ length of weir
$U =$ vapour rate, ft$^2$/sec ft$^2$ of plate bubbling area
$W =$ outlet weir height, in.

Operating limits for equation (9) are $L = 10–50$;
$U = 1.5–4.8; W = 2–4$.  

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Froth-height data obtained for plates with 2-, 3- and 4-in. outlet weirs were correlated as follows:

\[ h_f = 0.116 + 0.00395L + 0.088W + 0.0384U \]  
(10)

Operating limits for equation (10) are as follows: 

\[ L = 10-50; \ U = 2-6.5; \ W = 2-4. \]

APPENDIX

The eddy diffusion concept of liquid mixing has been applied to distillation trays by Anderson [9], Ruckenstein [8] and Andrew et al. [10].

Under steady-state conditions any concentration gradient which exists in the liquid on the plate must do so as the result of the differences between the rate of removal of volatile components by the stripping gas, the rate of replenishment by the inflowing liquid and a distribution effect due to liquid mixing. This is shown in Fig. (A.1) where conditions apply to a small vertical section of aerated liquid on a bubble plate.

Equating input to section shown in Fig. A.1 to output leads to the relation

\[ \frac{D_E}{d^2 x/dz^2} + \frac{L'}{x_1} \frac{d x}{d z} + \frac{y_1 - y_2}{h_L} = 0 \]  
(A.1)

Froth velocity \( V = \frac{L'}{h_L} = \frac{Q_L}{A \rho_f \rho_f} \)

where \( Q_L \) = liquid rate ft\( ^3 \)/sec per ft length weir

\( h_L \) = clear liquid height on plate, ft

Also if \( z/z_1 = w \), equation (A.1) becomes

\[ \frac{D_E}{V z_1} \frac{d^2 x}{d w^2} - \frac{d x}{d w} + (y_1 - y_2)(G/L) = 0 \]  
(A.2)

If the point efficiency is constant and the equilibrium relationship is linear, the equation reduces to

\[ \frac{1}{N_{pe}} \frac{d^2 x}{d w^2} - \frac{d x}{d w} = 0 \]  

(A.3)

where \( N_{pe} \) is the Peclet number \( (V z_1/D_E) \); \( x_e^* = (y_1 - b)/m \) and \( \lambda = m G/L' \). The Peclet group is the ratio of the tray length \( z_1 \) to the mixing length \( D_E/V \). A method of measuring the \( D_E \) values without resorting to mass transfer experiments was proposed by Brown [16] and Wharton [17]. It may be seen that with no material transfer between the gas and liquid phases, the diffusion equation (A.3) reduces to

\[ \frac{1}{N_{pe}} \frac{d^2 x}{d w^2} = 0 \]  

(A.4)

If the tray has a sufficient length of liquid travel so that none of the tracer reaches the tray inlet position, the solution of equation (A.4) is

\[ \frac{x - x_0}{x_e^* - x_0} = \frac{\exp(N_{pe} w) - 1}{\exp(N_{pe}) - 1} \]  

(A.5)

where \( x \) is the tracer concentration at any point \( w \), \( x_e^* \) is the tracer concentration at the injector grid and \( x_0 \) is the tracer concentration of the liquid fed

---

**Fig. A.1.**
The evaluation of liquid mixing effects on a sieve plate using unsteady and steady state tracer techniques

to the tray. If \( N_{pe} \gg 1 \) and except in the neighbourhood of \( w = 0 \)

\[
\frac{x - x_0}{x_0 - x_0} = \text{exp}[N_{pe}(w - 1)] \quad (A.6)
\]

By plotting values of \((x - x_0)/(x_0 - x_0)\) as a function of \((L - w)\) on semilogarithmic co-ordinates, the slope of the linear plot gives the value of the Peclet number \( N_{pe} \) from which \( D_E \) may be obtained.

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Notation

\[
\begin{array}{ll}
A & \text{Cross-sectional area of froth} \quad \text{ft}^2 \\
C & \text{Concentration of dye on plate at sampling point} \quad \text{lb/ft}^3 \\
C_0 & \text{Initial concentration of dye} \quad \text{lb/ft}^3 \\
C_1C_2 & \text{Constants in equation (6)} \\
D_E & \text{Longitudinal eddy diffusion coefficient of mixing} \quad \text{ft/sec} \\
E_{thr} & \text{Murnpree point efficiency} \\
G & \text{Gas rate} \quad \text{lb moles/sec} \\
h & \text{Space occupied by dye at zero time} \quad \text{ft} \\
h_f & \text{Foam height} \quad \text{ft} \\
h_L & \text{Clear liquid hold-up} \quad \text{ft} \\
l & \text{Distance to sampling point} \quad \text{ft} \\
L & \text{Liquid rate} \quad \text{gal/min ft length of weir} \\
L' & \text{Liquid rate} \quad \text{lb moles/sec} \\
N_{pe} & \text{Peclet number} \quad Vz_1/D_E \\
Q_L & \text{Liquid rate} \quad \text{ft}^3/\text{sec per ft of weir length} \\
t & \text{Time} \quad \text{secs} \\
u & \text{Vapour rate} \quad \text{ft}^3/\text{sec ft}^2 \text{ of plate bubbling area} \\
W & \text{Froth velocity} \quad \text{ft/sec} \\
W' & \text{Outlet weir height} \quad \text{in.} \\
x & \text{Liquid concentration of the sample} \quad \text{mole fraction or lb/ft}^3 \\
x_0 & \text{Composition of the liquid in equilibrium with the vapour entering the tray} \quad \text{mole fraction or lb/ft}^3 \\
x_e & \text{Salt concentration at the injector grid} \quad \text{mole fraction or lb/ft}^3 \\
x_0 & \text{Salt concentration of the mains water} \quad \text{mole fraction or lb/ft}^3 \\
y_1 & \text{Inlet gas concentration} \quad \text{mole fraction} \\
y_2 & \text{Outlet gas concentration} \quad \text{mole fraction} \\
z & \text{Distance from the inlet weir to the sampling point} \quad \text{ft} \\
z_1 & \text{Total tray length} \quad \text{ft} \\
p_f & \text{Froth density} \quad \text{ft}^3 \text{ of liquid/ft}^3 \text{ of froth} \\
p_i & \text{Clear liquid density} \quad \text{lb mole/ft}^3 \\
\lambda & \text{Ratio of the slope of the equilibrium line to the slope of the operating line, mg/L} \\
\end{array}
\]

References


Résumé—Un tamis rectangulaire de 5,85 m de long, 13,5 m de large, ayant des trous de \( \frac{1}{8} \) in. de diamètre et \( \frac{1}{16} \) in. de maille, a été utilisé pour évaluer les effets de mélange de liquides dans les colonnes à plateaux de ce type.

Le degré de mélange a été mesuré en fonction du coefficient tourbillonnaire longitudinal \((D_E)\) par une technique de traçage non stationnaire.

Pour le système air-eau on a fait une série d'expériences avec des retentions variant entre \( \frac{1}{4} \) et \( \frac{3}{4} \) in.; et des vitesses d'air variant entre \( 1 \) et \( 4 \) ft/sec. Pour des retentions de \( \frac{1}{4} \) et de \( 1 \) in. les valeurs de \( D_E \) sont reliées aux oscillations verticales du liquide créées par une vitesse de l'air d'environ \( 3,0 \) ft/sec. On constate que les oscillations réduisent l'effet de mélange.