Homework Set No. 2 ChE 363 Summer 2013 R.B. Eldridge

Diffusion Parameter Calculations

1) Work Problem 3.9
2) Work Problem 3.11
3) Work Problem 3.16
4) Estimate the diffusion coefficient of methyl chloride in sulfur dioxide at 1.0 atm and 50 C. Dipole moments for methyl chloride = 1.9 debye and for sulfur dioxide = 1.6 debye.

Steady State Diffusion Problems

5) Work Problem 3.28
6) Work Problem 3.33
7) Work Problem 3.34

Nonsteady State Problems

8) Work Problem 3.25
9) Black flag has an enhanced system for determining the efficiency of their new pesticides. They place a calibrated test spider at the start of a sealed path and have the spider walk a straight line until it falls over dead. The distance traveled is a direct measure of the lethality of the new pesticide. The following test is conducted. Determine the distance the spider travels prior to succumbing.

\[ T_{\text{Air}} = 30 \, ^\circ\text{C} \]
\[ P = 1 \, \text{Atm} \]
\[ \text{Air density} = 1.2 \, \text{kg/m}^3 \]
\[ \text{Air viscosity} = 1.8 \times 10^{-5} \, \text{kg/m sec} \]

\[ \text{Velocity of air / pesticide mixture over spider (} V_g \text{)} = 1.0 \, \text{m/sec} \]
\[ \text{Velocity of spider (} V_{\text{Bug}} \text{)} = 0.5 \, \text{cm/sec} \]
\[ \text{Diameter of test spider} = 1.0 \, \text{cm} \]
\[ \text{Concentration of pesticide in test cell} = 1.0 \times 10^{-3} \, \text{kgmol/m}^3 \]
\[ \text{Lethal concentration at center of bug} = 0.2 \times 10^{-3} \, \text{kgmol/m}^3 \]
\[ \text{Molecular weight of the test pesticide} = 200.0 \, \text{kg/kgmol} \]

Dr. Ima Bugg (Black Flag’s senior research engineer) has developed the following data.

\[ \text{Equilibrium data - } C_{A,\text{bug}} = 56.0 \, C_{A,\text{air}} \]

The mass transfer coefficient is determined from the following relationship:

\[ \text{Sh} = 0.003 \, (Re_p)^{0.75} \, (Sc)^{0.33} \]
\[ \text{Re}_g = \frac{d_{\text{bug}} V_g \rho}{\mu} \]
\[ \text{Sc} = \frac{\mu}{\rho D_{AB}} \]
\[ \text{Sh} = \frac{k_c d_{\text{bug}}}{D_{AB}} \]

Previous tests have determined that the gas phase diffusion coefficient is \(0.20 \text{ cm}^2/\text{sec}\) at 25 C and that the diffusion coefficient in the bug body is \(1/200\) of the gas value.
Exercise 3.9

Subject: Estimation of the binary gas diffusivity for nitrogen (A) - toluene (B) at 25°C (298 K) and 3 atm

Assumptions: No need to correct estimated diffusivity for high pressure with Takahashi method.

Find: Binary gas diffusivity using the method of Fuller, Shettler, and Giddings.

Analysis: Use Eq. (3-36), with,

\[ M_{AB} = \frac{2}{1} - \frac{1}{1} = 42.9 \]

\[ \frac{28}{92} \]

From Table 3.1,

\[ \sum V_A = 18.5, \quad \sum V_B = 7(15.9) + 8(2.31) - 18.3 = 111.5 \]

\[ D_{AB} = \frac{0.00143(298)^{1.75}}{(3)(42.9)^{1/2}[18.5^{1/3} + 111.5^{1/3}]^2} = 0.028 \text{ cm}^2/\text{s} \]

Exercise 3.10

Subject: Correction of gas binary diffusivity for high pressure.

Given: Results of Example 3.3 for oxygen-benzene system at 38°C (311 K) and 2 atm, which give, \( D_{AB} = 0.0494 \text{ cm}^2/\text{s} \)

Find: Diffusivity at 100 atm.

Analysis: If the Eq. (3-36) is applied,

\[ D_{AB} = 0.0494 \frac{3}{100} = 0.00148 \text{ cm}^2/\text{s} \]

Apply the Takahashi correlation of Fig. 3.3, based on reduced \( T \) and \( P \).
For equimolar mixture, \( T^* = T/T_c \) and \( P^* = P/P_c \)
where, \( T_c = 0.5(154 + 563) = 359 \text{ K} \) and \( P_c = 0.5(48.6 + 49.7) = 49.1 \text{ atm} \)
Therefore, \( T^* = 311/359 = 0.866 \), and \( P^* = 100/49.1 = 2.04 \)

We are outside the range of the Takahashi correlation, but it appears that the correction would greatly decrease the diffusivity, by a factor of 10 or more.
Exercise 3.11

Subject: Estimation of infinite-dilution liquid diffusivity for carbon tetrachloride at 25°C (298 K) in four different solvents.

Given: Experimental values diffusivity for solvents of (a) methanol, (b) ethanol, (c) benzene, and (d) n-hexane.

Find: Diffusivities by the methods of Wilke and Chang (W-C), and of Hayduk and Minhas (H-M). Compare predicted values to given experimental values.

Analysis: Let A = the solute, CCl₄, and B = a solvent. The Wilke-Chang equation, Eq. (3-39), is

\[ D_{AB} = \frac{7.4 \times 10^{-4} (\phi_B M_B)^{1/2} T}{\mu_B \nu_A^{0.4}} \]  

(1)

From Table 3.2, \( \nu_A = 14.8+4(21.6)=101.2 \) cm³/mol
Using Eq. (1) with the following parameters, values of diffusivity are computed.

<table>
<thead>
<tr>
<th>Solvent, B</th>
<th>( M_B )</th>
<th>( \phi_B )</th>
<th>( \mu_B ), cP</th>
<th>( D_{AB} ) (W-C)</th>
<th>( D_{AB} ) (Expt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methanol</td>
<td>32</td>
<td>1.9</td>
<td>0.57</td>
<td>1.89 x 10⁻⁵</td>
<td>1.69 x 10⁻⁵</td>
</tr>
<tr>
<td>Ethanol</td>
<td>46</td>
<td>1.5</td>
<td>1.17</td>
<td>0.98 x 10⁻⁵</td>
<td>1.50 x 10⁻⁵</td>
</tr>
<tr>
<td>Benzene</td>
<td>78</td>
<td>1.0</td>
<td>0.60</td>
<td>2.03 x 10⁻⁵</td>
<td>1.92 x 10⁻⁵</td>
</tr>
<tr>
<td>n-Hexane</td>
<td>86</td>
<td>1.0</td>
<td>0.32</td>
<td>4.00 x 10⁻⁵</td>
<td>3.70 x 10⁻⁵</td>
</tr>
</tbody>
</table>

Except for ethanol, the Wilke-Chang equation makes good predictions.

The applicable Hayduk-Minhas equation, Eq. (3-42), is,

\[ D_{AB} = 155 \times 10^{-4} \frac{T^{1.29} (\overline{D}_B^{0.5} / \overline{D}_A^{0.42})}{\mu_B^{0.9} \nu_B^{0.23}} \]  

(2)

where, for the nonpolar solute, CCl₄, with methanol and ethanol solvents, both \( \overline{D}_B \) and \( \nu_B \) must be multiplied by 8\( \mu_B \) in cP. Parachors for methanol and benzene are obtained from Table 3.3. Parachors for ethanol, n-Hexane, and carbon tetrachloride are obtained by structural contributions from Table 3.4. Using Eq. (2) with \( \overline{D}_A = 229.8 \) and the following parameters, values of diffusivity are computed.

<table>
<thead>
<tr>
<th>Solvent, B</th>
<th>( \overline{D}_B )</th>
<th>( \nu_B )</th>
<th>( \mu_B ), cP</th>
<th>( D_{AB} ) (H-M)</th>
<th>( D_{AB} ) (Expt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methanol</td>
<td>88.8</td>
<td>37.0</td>
<td>0.57</td>
<td>2.55 x 10⁻⁵</td>
<td>1.69 x 10⁻⁵</td>
</tr>
<tr>
<td>Ethanol</td>
<td>125.3</td>
<td>59.2</td>
<td>1.17</td>
<td>1.70 x 10⁻⁵</td>
<td>1.50 x 10⁻⁵</td>
</tr>
<tr>
<td>Benzene</td>
<td>205.3</td>
<td>96.0</td>
<td>0.60</td>
<td>1.96 x 10⁻⁵</td>
<td>1.92 x 10⁻⁵</td>
</tr>
<tr>
<td>n-Hexane</td>
<td>271.0</td>
<td>140.6</td>
<td>0.32</td>
<td>3.05 x 10⁻⁵</td>
<td>3.70 x 10⁻⁵</td>
</tr>
</tbody>
</table>

Except for methanol and n-hexane, the predictions by the Hayduk-Minhas equation are good.
Exercise 3.16

Subject: Liquid diffusivities for the ethanol (A)-benzene (B) system at 45°C (318 K) over the entire composition range.

Given: Experimental activity coefficients in Exercise 2.23.

Find: Effect of composition on diffusivities of both ethanol and benzene.

Analysis: Use Eq. (3-42) of Hayduk-Minas to estimate the infinite-dilution liquid diffusivities, with liquid viscosities from Perry's Handbook. From Table 3.3, the parachor of benzene is given. The parachor for ethanol is estimated from Table 3.4. Table 3.2 is used to estimate molecular volumes. The resulting parameters are as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>( \mathcal{P} )</th>
<th>( \nu )</th>
<th>( \mu ), cP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>205.3</td>
<td>96</td>
<td>0.48</td>
</tr>
<tr>
<td>Ethanol</td>
<td>125.3</td>
<td>59.2</td>
<td>0.79</td>
</tr>
</tbody>
</table>

For benzene at infinite dilution in ethanol, values of molecular volume and the parachor for ethanol must be multiplied by 8 times the viscosity of ethanol. Thus, from Eq. (3-42), using \( \mathcal{P} = 8(0.79)(125.3) = 792 \) and \( \nu = 8(0.79)(59.2) = 374 \) for ethanol, the solvent in this case,

\[
(D_{\text{BA}})_{\infty} = 1.55 \times 10^{-4} \frac{318.29 (792^{0.5} / 205.3^{0.42})}{0.79^{0.82} (374)^{0.23}} = 2.51 \times 10^{-4} \text{ cm}^2 / \text{s}
\]

For ethanol at infinite dilution in benzene, Eq. (3-42) gives,

\[
(D_{\text{AB}})_{\infty} = 1.55 \times 10^{-4} \frac{318.29 (205.3^{0.5} / 125.3^{0.42})}{0.48^{0.82} (90)^{0.23}} = 3.4 \times 10^{-5} \text{ cm}^2 / \text{s}
\]

Use Eqs. (3-51) and (3-52) of Vignes to compute \( D_{\text{AB}} \) and \( D_{\text{BA}} \) as a function of composition,

\[
D_{\text{AB}} = (D_{\text{AB}})_{\infty} (D_{\text{BA}})_{\infty} \left( 1 + \frac{\partial \ln \gamma_A}{\partial \ln x_A} \right)_{T,P} \quad (1)
\]

\[
D_{\text{BA}} = (D_{\text{BA}})_{\infty} (D_{\text{AB}})_{\infty} \left( 1 + \frac{\partial \ln \gamma_B}{\partial \ln x_B} \right)_{T,P} \quad (2)
\]

The partial derivatives are evaluated numerically with,

\[
\frac{\partial \ln \gamma_A}{\partial \ln x_A} = \frac{\Delta \ln \gamma_A}{\Delta \ln x_A} \quad \text{and} \quad \frac{\partial \ln \gamma_B}{\partial \ln x_B} = \frac{\Delta \ln \gamma_B}{\Delta \ln x_B}
\]
Exercise 3.16 (continued)

Analysis: (continued)

Using the data from Exercise 2.23,

<table>
<thead>
<tr>
<th>$x_A$</th>
<th>$\ln \gamma_A$</th>
<th>$\ln \gamma_B$</th>
<th>$\frac{\Delta \ln \gamma_A}{\Delta \ln x_A}$</th>
<th>$\frac{\Delta \ln \gamma_B}{\Delta \ln x_B}$</th>
<th>$D_{A,B}$, cm$^2$/s</th>
<th>$D_{B,A}$, cm$^2$/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>2.0937</td>
<td>0.0220</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0374</td>
<td>2.0937</td>
<td>0.0220</td>
<td>-0.501</td>
<td>-0.466</td>
<td>3.4x10^{-5}</td>
<td></td>
</tr>
<tr>
<td>0.0673</td>
<td>1.6153</td>
<td>0.0519</td>
<td>-0.773</td>
<td>-0.757</td>
<td>1.66x10^{-5}</td>
<td>1.78x10^{-5}</td>
</tr>
<tr>
<td>0.0972</td>
<td>0.7090</td>
<td>0.2599</td>
<td>-0.785</td>
<td>-0.783</td>
<td>0.727x10^{-9}</td>
<td>0.777x10^{-3}</td>
</tr>
<tr>
<td>0.2057</td>
<td>0.3136</td>
<td>0.5392</td>
<td>-0.664</td>
<td>-0.651</td>
<td>0.645x10^{-4}</td>
<td>0.651x10^{-4}</td>
</tr>
<tr>
<td>0.3141</td>
<td>0.1079</td>
<td>0.8645</td>
<td>-0.414</td>
<td>-0.353</td>
<td>0.949x10^{-5}</td>
<td>0.985x10^{-5}</td>
</tr>
<tr>
<td>0.4170</td>
<td>0.0002</td>
<td>1.3177</td>
<td>-0.186</td>
<td>-0.121</td>
<td>1.56x10^{-5}</td>
<td>1.72x10^{-5}</td>
</tr>
<tr>
<td>0.5199</td>
<td>-0.0077</td>
<td>1.3999</td>
<td></td>
<td></td>
<td>2.08x10^{-5}</td>
<td>2.25x10^{-5}</td>
</tr>
<tr>
<td>0.6143</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.51x10^{-5}</td>
</tr>
</tbody>
</table>

These results give the following plot showing that the liquid diffusivities do not vary linearly with composition.
Problem 4

Estimate the diffusion coefficient for a mixture of methyl chloride (MC) and sulfur dioxide (SD) at 1 atm and 50°C. Data needed to employ Brokaw’s relation are shown below:

<table>
<thead>
<tr>
<th></th>
<th>Methyl Chloride</th>
<th>Sulfur Dioxide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole moments, debyes</td>
<td>1.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Liquid molar volume at $T_b$, cm$^3$/g-mol</td>
<td>50.6</td>
<td>43.8</td>
</tr>
<tr>
<td>Normal boiling point, K</td>
<td>249</td>
<td>263</td>
</tr>
</tbody>
</table>

**Solution:** With Eqs. (11-3.9) and (11-3.12)

\[
\delta_{MC} = \frac{(1.94 \times 10^3)(1.9)^2}{(50.6)(249)} = 0.55
\]

\[
\delta_{SD} = \frac{(1.94 \times 10^3)(1.6)^2}{(43.8)(263)} = 0.43
\]

\[
\delta_{MC-SD} = [(0.55)(0.43)]^{1/2} = 0.490
\]

Also, with Eqs. (11-3.10) and (11-3.13),

\[
\frac{\sigma_{MC}}{k} = 1.18[1 + 1.3(0.55)^2](249) = 409K
\]

\[
\frac{\sigma_{SD}}{k} = 1.18[1 + 1.3(0.43)^2](263) = 385K
\]

\[
\frac{\sigma_{MC-SD}}{k} = [(409)(385)]^{1/2} = 397K
\]

Finally, with Eqs. (11-3.11) and (11-3.14),

\[
\delta_{MC} = \left[ \frac{(1.585)(50.6)}{1 + (1.3)(0.55)^2} \right]^{1/3} = 3.86\text{Å}
\]

\[
\delta_{SD} = \left[ \frac{(1.585)(43.8)}{1 + (1.3)(0.43)^2} \right]^{1/3} = 3.82\text{Å}
\]

\[
\delta_{MC-SD} = [(3.86)(3.82)]^{1/2} = 3.84\text{Å}
\]
To determine $\Omega_D$, $T^* = kT / C_{AB} = 3223/397 = 0.814$. With Eq. (11-3.6), $\Omega_D = 1.598$. Then with Eq. (11-3.8), to correct this value

$$\Omega_D = 1.598 + \frac{(0.19)(0.490)^2}{0.814} = 1.654$$

With Eq. (11-3.2) and $M_{MC} = 50.488$, $M_{SD} = 64.063$,

$$D_{MC-SD} = \frac{(1.858 \times 10^{-3})(323^{3/2})}{(50.488 + 64.063)^{1/2}} \left[ \frac{50.488 + 64.063}{(50.488)(64.063)} \right]^{1/2} \left(\frac{1}{(1)(3.84)^2(1.654)} \right) = 0.083 \text{cm}^2 / \text{s}$$

The experimental value is 0.077 cm$^2$/s and the estimation error 8 percent.

**Discussion:** A comprehensive review of the theory and available data for gas-diffusion coefficients is now available [142]. There have been numerous studies covering wide temperature ranges, and the applicability of Eq. (11-3.1) is well verified. Most investigators select the Lennard-Jones potential for its convenience and simplicity. The difficult task is to locate appropriate values of $\sigma$ and $\epsilon$. A few approximate estimation techniques are described in Sec. 2-7, and a tabulation is shown in Appendix C. Brokaw also suggests other relations, e.g., Eqs. (11-3.10) and (11-3.11). Even after the pure-component values of $\sigma$ and $\epsilon$ have been selected, a combination rule is necessary to obtain $\sigma_{AB}$ and $\epsilon_{AB}$. Most studies have employed Eqs. (11-3.4) and (11-3.5), as they are simple and theory suggests no particularly better alternatives.

It is important to employ values of $\sigma$ and $\epsilon$ obtained from the same source. Published values of these parameters differ considerably, but $\sigma$ and $\epsilon$ from a single source will often lead to the same result as the use of a quite different pair from another source.

Besides the few estimation schemes described above, there are offshoots, partly empirical in nature, that have been suggested for obtaining better agreement with data [143, 152].
Exercise 3.28

Subject: Evaporation of water (A) from a film on a flat plate into air (B) at 1 atm (101.3 kPa) and 25°C (298 K) that is flowing past the plate.

Given: Free-stream air velocity = \( u_a = 2 \) m/s. Plate length = \( L = 2 \) inches = 0.0508 m. Diffusivity of water vapor in air = \( D_{AB} = 0.25 \) cm\(^2\)/s = 0.25 \( \times 10^{-4} \) m\(^2\)/s


Find: Evaporation flux at a distance of 50% of \( L = x = 0.0254 \) m from the leading edge of the plate.

Analysis: First check to see if the boundary layer is laminar. From Perry's Handbook, for air at 25°C, \( \mu = 0.018 \) cp = 1.8 \( \times 10^{-5} \) kg/m-s. From the ideal gas law, \( \rho = PM/RT = (101.3)(29)/(8.314)(298) = 1.186 \) kg/m\(^3\)

From Eq. (3-127), at \( x = 0.0254 \) m,

\[
N_{Re} = \frac{\sqrt{\rho \mu}}{x} = \frac{0.0254(2)(1.186)}{1.8 \times 10^{-5}} = 3,350
\]

Therefore, the boundary layer is laminar, since the criterion is less than 5 \( \times 10^3 \).

Since the liquid film is pure water, the only mass-transfer resistance for evaporation of water into air is in the air boundary layer. From Eq. (3-105), the molar mass-transfer flux of water is,

\[
\frac{n_{H_2O}}{A} = k_{ir} \left[ (c_{H_2O})_i - (c_{H_2O})_o \right] 
\]

(1)

For a laminar boundary layer, the local mass-transfer coefficient can be obtained from Eq. (3-132), which can be rearranged to give:

\[
k_{ir} = 0.332 \frac{D_{AB}}{x} \left( N_{Re} \right)^{1/2} \left( N_{Sc} \right)^{1/3} 
\]

(2)

From Eq. (3-101), \( N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{18 \times 10^{-5}}{(1.186)(0.25 \times 10^{-4})} = 0.607 \)

From Eq. (2), \( k_{ir} = 0.332 \frac{(0.25 \times 10^{-4})}{(0.0254)} (3,350)^{1/2}(0.607)^{1/3} = 0.16 \) m/s

At 25°C, vapor pressure of water = 0.46 psia. Therefore, mole fraction of water at the interface = \( (y_{H_2O})_i = 0.46/14.7 = 0.0313 \) \( (c_{H_2O})_i = (y_{H_2O}) \rho/M_{sw} = 0.0313(1.186)/29 = 0.0128 \) kmol/m\(^3\)

From Eq. (1), \( \frac{n_{H_2O}}{A} = (0.016)[0.00128 - 0] = 2.05 \times 10^{-5} \) kmol / s - m\(^2\)
Exercise 3.33

Subject: Sublimation of a long circular cylinder of naphthalene (A) to air (B) flowing normal to it.

Given: Long, circular cylinder of 1-inch (0.0254 m) diameter, D. Air at 100°C (373 K) and 1 atm flows at a Reynolds number, \( N_{Re} \), of 50,000. Physical properties given in Example 3.14.

Assumptions: Negligible bulk flow effect.

Find: Average sublimation flux in kmol/s-m².

Analysis: Only mass-transfer resistance is in the air. From Eq. (3-105), the flux is,

\[
n_A / A = N_A = k_e (c_A - c_{Ae})
\]

(1)

Using the Chilton-Colburn analogy, Eq. (3-169) applies, which combined with Eq. (3-165) gives,

\[
k_e = \frac{0.026G}{\rho^{0.195}} (N_{Re})^{2/3} \quad (N_{Sc})^{2/3}
\]

(2)

From the Reynolds number, mass velocity, \( G \), is, using \( \mu = 0.0215 \text{ cP} \) (2.15 x 10⁻⁴ kg/s-m²),

\[
G = \frac{N_{Re} H}{D^2} = \frac{(50,000)(2.15 \times 10^{-4})}{0.0254} = 42.3 \text{ kg/s-m²}
\]

From Example 3.14, gas density = 0.0327(29) = 0.948 kg/m³ and \( N_{Sc} = 2.41 \).

From Eq. (2),

\[
k_e = \frac{0.0266G}{\rho^{0.195}} (N_{Re})^{2/3} = \frac{0.0266(42.3)(50,000)^{0.195}}{0.948 (2.41)^{2/3}} = 0.080 \text{ m/s}
\]

Using properties in Example 3.14,

\[
c_A = y_A c = \frac{P_A}{P} c = \frac{10}{760} (0.0327) = 4.3 \times 10^{-4} \text{ kmol/m³} \quad \text{and} \quad c_{Ae} = 0
\]

From Eq. (1),

\[
N_A = k_e (c_A - c_{Ae}) = (0.080)(4.3 \times 10^{-4} - 0) = 3.44 \times 10^{-5} \text{ kmol/s-m²}
\]
Exercise 3.34

Subject: Sublimation of a sphere of naphthalene (A) into air (B) flowing past it.

Given: Sphere of 1-inch (0.0254 m) diameter, $D$. Air at 100°C (373 K) and 1 atm flows at a Reynolds number, $N_{Re}$, of 50,000. Physical properties given in Example 3.14.

Assumptions: Negligible bulk flow effect.

Find: Initial average sublimation flux in kmol/s-m² and comparison to that in a bed packed with the same spheres at a void fraction of 0.5.

Analysis: Only mass-transfer resistance is in the air. From Eq. (3-105), the flux is,

$$n_A / A = N_A = k_c (c_A - c_{A_e})$$

(1)

Using the Chilton-Colburn analogy, Eq. (3-170) applies, which combined with Eq. (3-165) gives,

$$k_c = \frac{0.37G}{\rho} \left( \frac{N_{Re}}{N_{Sc}} \right)^{0.4}$$

(2)

From the Reynolds number, mass velocity, $G$, is, using $\mu = 0.0215$ cP (2.15 x 10⁻³ kg/s-m²),

$$G = \frac{N_{Re} \mu}{D} = \frac{(50,000)(2.15 \times 10^{-3})}{0.0254} = 42.3 \text{ kg/s/m}^2$$

From Example 3.14, gas density = 0.0327(29) = 0.948 kg/m³ and $N_{Sc} = 2.41$.

From Eq. (2),

$$k_c = \frac{0.37(42.3)}{0.948} \left( \frac{50,000}{(2.41)^{2/3}} \right)^{0.4} = 0.121 \text{ m/s}$$

Using properties in Example 3.14,

$$c_{A_e} = \gamma_A c = \frac{P^A}{P} c = \frac{10}{760} (0.0327) = 4.3 \times 10^{-4} \text{ kmol/m}^3 \text{ and } c_{A_e} = 0$$

From Eq. (1),

$$N_A = k_c (c_A - c_{A_e}) = (0.121)(4.3 \times 10^{-4} - 0) = 5.20 \times 10^{-5} \text{ kmol/s-m}^2$$

From Eq. (3-171), packed bed correlation can't used with $N_{Re} = 50,000$. 
Exercise 3.25

Subject: Diffusion of moisture from a 2-inch (5.08 cm) diameter (a = 2.54 cm radius) spherical ball of clay into air

Given: Initial moisture content of clay = 10 wt%. Equilibrium moisture content of clay-air interface = 3 wt%. Diffusivity of water in clay = D = 5 x 10^{-6} cm^2/s.

Assumptions: Diffusivity is uniform throughout clay.

Find: Time for average moisture content to drop to 5 wt%.

Analysis: By definition in Eq. (3-80), the unaccomplished fractional moisture concentration change after time, \( t \) is
\[
E_{\text{avg}} = \frac{(c_t - c_{\text{avg}})}{(c_i - c_e)} = (3-5)/(3-10) = 0.286
\]
The lower curve in Fig. 3.9 relates that change to the Fourier number. From the figure,
\[
N_{Fo} = 0.08 = \frac{D t}{a^2} = \frac{(5 \times 10^{-6})}{2.54^2} \rightarrow t = 7.75 \times 10^{-7} t
\]

Solving Eq. (1), gives \( t = 1.03 \times 10^3 \) s = 28.7 h
Problem 11

Mass Transfer Properties

\[ E = 1.2 \text{ kg/m}^3, \mu_{\text{air}} = 1.8 \times 10^{-5} \text{ kg/m} \cdot \text{s} \]

\[ \text{Re} = \frac{(0.01m)(1.0 \text{ m/sec})(1.2 \text{ kg/m}^2)}{1.8 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 666.7 \]

\[ \text{Sc} = \frac{\mu}{eD_{AB}} = \frac{(1.8 \times 10^{-5} \text{ kg/m} \cdot \text{s})}{1.2 \text{ kg/m}^2 (2.061 \times 10^{-5} \text{ m}^2 / \text{s})} \]

\[ D_{AB} = \left( \frac{273.2 + 30}{273.2 + 25} \right)^{0.75} \left( \frac{0.20 \text{ cm}^2}{\text{sec}} \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^{1} = 2.061 \times 10^{-5} \text{ m}^2 / \text{sec} \]

\[ \text{Sc} = 0.727 \]

\[ \text{Sh} = \frac{k_c d}{D_{AB}} = 0.0030(666.7)^{0.75}(0.727)^{0.33} = 0.3559 \]

\[ k_c = 0.3559(2.061 \times 10^{-5} \text{ m}^2 / \text{sec}) = 0.00072 \text{ m/sec} \]

Sphere

\[ \alpha = \frac{k_c d}{K D_{\text{eff}}} \]

\[ D_{\text{eff}} = \frac{1}{200}(2.061 \times 10^{-5} \text{ m}^2 / \text{sec}) = 1.030 \times 10^{-7} \text{ m}^2 / \text{sec} \]

\[ k_c = 0.00145 \text{ m}^2 / \text{sec}, \quad d = 0.01 \text{ m} \]

\[ K = 56.0 \]

\[ \alpha = \left( \frac{0.00072 \text{ m/sec}}{56.0} \right) \left( \frac{0.01 \text{ m}}{1.030 \times 10^{-7} \text{ m}^2 / \text{sec}} \right) \]

\[ \alpha = 1.25 \quad \text{at } r = 0 \]

\[ C_{A0} = 0 \quad \frac{1}{\alpha} \cong 0.80 \]

\[ C_A = 0.2 \times 10^{-3} \text{ kgmol/m}^3 \]

\[ C_{A0} = 0 \]

\[ C_{A1}^* = 56.0 (1 \times 10^{-3}) = 0.056 \text{ kgmol/m}^3 \]
\[
\left( \frac{0.0002 - 0.056}{0 - 0.056} \right) = 0.996
\]

\[
\left( \frac{D_{\text{eff}}t}{R^2} \right) \geq 0.15
\]

\[
t = \frac{0.15 \left( \frac{0.01m}{2} \right)^2}{(1.017 \times 10^{-7} m^2 / s)} = 36.0 \text{ sec}
\]

**Distance traveled**

36.0 sec (0.5 cm/sec) = 18.4 cm
Non-steady State Mass Transfer Tutorial Problem

Black flag has an enhanced system for determining the efficiency of their new pesticides. They place a calibrated test spider at the start of a sealed path and have the spider walk a straight line until it falls over dead. The distance traveled is a direct measure of the lethality of the new pesticide. The following test is conducted. Determine the distance the spider travels prior to succumbing.

\[ T_{\text{Air}} = 30 \, ^\circ\text{C} \]
\[ P = 1 \, \text{Atm} \]
\[ \text{Air density} = 1.2 \, \text{kg/m}^3 \]
\[ \text{Air viscosity} = 1.8 \times 10^{-5} \, \text{kg/m sec} \]

Velocity of air / pesticide mixture over spider \( (V_g) = 1.0 \, \text{m/sec} \)
Velocity of spider \( (V_{\text{bug}}) = 0.5 \, \text{cm/sec} \)
Diameter of test spider = 1.0 cm
Concentration of pesticide in test cell = 1.0 E-3 kgmol/m^3
Lethal concentration at center of bug = 0.020 kgmol/m^3
Molecular weight of test pesticide = 200.0 kg/kgmol

Dr. Ima Bugg (Black Flag's senior research engineer) has developed the following data.

Equilibrium data - \( C_{A,\text{bug}} = 56.0 \, C_{A,\text{air}} \)

The mass transfer coefficient is determined from the following relationship:

\[ S_h = 0.003 \, (Re_g)^{0.75} \, (Sc)^{0.33} \]

\[ Re_g = d_{\text{bug}} \, V_g \, \rho \, / \, \mu \]

\[ Sc = \mu \, / \, \rho \, D_{AB} \]

\[ Sh = k_c \, d_{\text{bug}} \, / \, D_{AB} \]

Previous tests have determined that the gas phase diffusion coefficient is 0.20 cm^2/sec at 25 C and that the diffusion coefficient in the bug body is 1/200 of the gas value.
**Tutor 101**

**Problem**

\[
Re = \frac{0.01 \text{m} \left( 1.0 \text{ m}^3 \right) \left( 1.2 \text{ kg/m}^3 \right)}{1.4 \times 10^{-5} \text{ m/s}} = 666.7
\]

\[
Sc = 0.727
\]

\[
Sh = \frac{k_e L}{D_{AB}} = 0.0080 \left( 666.7 \right)^{0.75} \left( 0.727 \right)^{0.553} = 0.3559
\]

\[
k_e = \frac{0.3559 \left( 2.061 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \right)}{0.01 \text{ m}} = 0.00072 \text{ m/s}
\]

\[
\alpha = \frac{k_e \nu}{k_{AB}} =
\]

\[
D_{AB} = \frac{2.061 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}}{200} = 1.070 \times 10^{-7} \text{ m/s}
\]

\[
\gamma = \frac{0.01}{2} = 0.005 \text{ m}
\]

\[
\alpha = \frac{\left( 0.00072 \text{ m/s} \right) \left( 0.005 \text{ m} \right)}{56 \left( 1.030 \times 10^{-7} \text{ m}^2 \text{ s}^{-1} \right)} = 0.624
\]
\[ C_{A0} = 0 \quad C_{A} = 0.020 \ \frac{15 \text{ mol}}{m^3} \]
\[ C_{A}^* = 56 \ (1.0 \times 10^{-3} \ \frac{\text{kmol}}{m^3}) = 0.056 \ \frac{\text{kmol}}{m^3} \]
\[ Y_{\text{axis}} = \frac{0.020 - 0.056}{0 - 0.056} = 0.643 \]
\[ \frac{1}{\alpha} = 1.6 \]

From Figure \[ \frac{D}{\rho \cdot \beta} \ t = 0.90 \]
\[ t = 0.90 \left( \frac{0.01 \text{ m}}{2} \right)^2 = \frac{97.1 \text{ sec}}{1.07 \times 10^{-7} \frac{\text{m}^2}{\text{sec}}} \]

Distance \[ = 97.1 \text{ sec} \left( \frac{0.026 \text{ m}}{\text{sec}} \right) = 48.5 \text{ cm} \]
Figure 4-16 Unsteady concentration profiles in a sphere (from Boelter et al., 1965)

Figure 4-17 Unsteady mass transfer in a sphere as a function of $\alpha$ (from Crank, 1956)