### Important Heat Transfer Parameters

#### General Parameters:

- **q or \( \dot{Q} \)**
  \[ \frac{BTU}{hr} , W \] Heat transfer rate

- **\( q'' \) or \( \bar{q} \)**
  \[ \frac{BTU}{hr ft^2} , \frac{W}{m^2} \] Heat flux (per unit area)

- **\( C_p \)**
  \[ \frac{BTU}{lbm \cdot ^\circ F} \cdot \frac{I}{kgK} \] Specific heat capacity

- **\( k \)**
  \[ \frac{BTU}{hr ft^2 \cdot ^\circ F} \cdot \frac{W}{m^2 K} \] Thermal conductivity

- **\( h \)**
  \[ \frac{BTU}{hr ft^2 \cdot ^\circ F} \cdot \frac{W}{m^2 K} \] Convective heat transfer coefficient

- **\( \alpha = \frac{k}{\rho C_p} \)**
  \[ \frac{ft^2}{hr \cdot m^2 \cdot s} \] Thermal diffusivity

- **\( R \)**
  \[ \frac{hr ft^2 \cdot ^\circ F}{BTU} \cdot \frac{m^2 K}{W} \] Thermal resistance

- **\( U_o, U_i \)**
  \[ \frac{BTU}{hr ft^2 \cdot ^\circ F} \cdot \frac{W}{m^2 K} \] Overall heat transfer coefficient

  (based on outside or inside area respectively)

- **\( L_c \)**
  \[ ft, m \] Characteristic length

  (\( r \) for cylinder or sphere; \( \frac{w}{2} \) for slab)

#### Heat exchanger parameters:

- **\( \Delta T_{TM} = F_G \Delta T_{LM} \)**
  \[ ^\circ F, K \] True mean temperature difference

  Countercurrent: \( F_G = 1 \)

  Shell&Tube: \( F_G = f(Z, \eta_H) \leq 1 \)

  use plots for \( #_{shell\_passes} #_{tube\_passes} \) exchanger

- **\( \Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{ln(\Delta T_1/\Delta T_2)} \)**
  \[ ^\circ F, K \] Log mean temperature difference

  \( (\Delta T_1 = T_{fluid1,in} - T_{fluid2,out}; \Delta T_2 = T_{fluid1,out} - T_{fluid2,in}) \)

- **\( Z = \frac{T_{ha} - T_{hb}}{T_{cb} - T_{ca}} \)**
  \[ -\text{none}- \] Ratio of \( \Delta T_{shell} \) to \( \Delta T_{tubes} \)

  Subscripts: \( h \rightarrow \text{shell}; c \rightarrow \text{tube}; a \rightarrow \text{in}; b \rightarrow \text{out} \)

- **\( \eta_H = \frac{T_{cb} - T_{ca}}{T_{ha} - T_{ca}} \)**
  \[ -\text{none}- \] Heating effectiveness, ratio of \( \Delta T_{actual} \) to \( \Delta T_{max} \)

  Subscripts: \( h \rightarrow \text{shell}; c \rightarrow \text{tube}; a \rightarrow \text{in}; b \rightarrow \text{out} \)
Important dimensionless numbers:

\[ \text{Nu} = \frac{hD}{k_{\text{fluid}}} \quad \text{-none-} \quad \text{Nusselt number} \]
(Convevive transfer / conductive transfer)

\[ \text{Pr} = \left( \frac{C_p \mu}{k} \right)_{\text{fluid}} \quad \text{-none-} \quad \text{Prandtl number} \]
(momentum diffusivity / thermal diffusivity)

\[ \text{Bi} = \frac{hL_c}{k_{\text{body}}} \quad \text{-none-} \quad \text{Biot number} \]
(convection out / conduction through body)
\( \text{Bi} \ll 1 \Rightarrow \text{fast conduction, uniform T in body} \)
\( \text{Bi} \gg 1 \Rightarrow \text{fast convection, constant T at surface} \)

\[ \text{Fo} = \frac{a_{\text{body}} L_c}{T_c} \quad \text{-none-} \quad \text{Fourier number} \]
(conduction rate / storage rate)

\[ \Theta = \frac{T(t) - T_b}{T_0 - T_b} \quad \text{-none-} \quad \text{Dimensionless temperature (similarly for dimensional average temperature} \bar{\Theta} (\bar{T}(t))) \]
\( T_b = \text{bulk fluid; } T_0 = \text{initial in the solid} \)

\[ \Phi = \frac{\mu(T_{\text{bulk,ave}})}{\mu(T_{\text{wall,ave}})} \quad \text{-none-} \quad \text{Viscosity ratio (accounts for \mu change with T)} \]
Note: may need to iterate to find \( T_{\text{wall,ave}} \)
Calculating Heat Transfer Rates

*General expressions:*

\[ \dot{Q} = \dot{m}C_p\Delta T \]
Internal temperature change (\(\Delta T\) of stream)

\[ \dot{Q} = \frac{\Delta T_{a\rightarrow z}}{R_{a\rightarrow z}} \]
General heat transfer from point a to point z (\(\Delta T\) driving force)
There may be one or multiple resistances between a and z

\[ \dot{Q} = \frac{\Delta T_{TM}}{R_{total}} \]
Heat transfer in heat exchanger (\(\Delta T_{TM}\) is true mean driving force averaged between the pipe start and end)

\[ R_{total} = \frac{1}{U_oA_o} = \frac{1}{U_iA_i} \]
Overall resistance to heat transfer

\[ R_{total} = \sum_{\text{series}}^{a\rightarrow z} R \]
Resistances in series (one term per resistance in system)
Remember \(\dot{Q}\) is constant through all resistance layers in series!

\[ \frac{1}{R_{total}} = \sum_{\text{parallel}}^{a\rightarrow z} \frac{1}{R} \]
Resistances in parallel (one term per resistance in system)

*Forms of resistance terms – combine as needed for each of the system resistances to get \(R_{total}\):*

\[ R_{slab} = \frac{\Delta x}{kA} \]
Slab conduction resistance

\[ R_{cyl} = \frac{\Delta r}{kA_{LM}} = \frac{\ln(D_o/D_i)}{2\pi kL} \]
Cylindrical conduction resistance (using log-mean area)

\[ R_{sph} = \frac{\Delta r}{kA_{GM}} = \frac{(D_o-D_i)}{2\pi kL\sqrt{D_oD_i}} \]
Spherical conduction resistance (using geometric-mean area)

\[ R_{conv} = \frac{1}{hA} \]
Convection resistance

\[ R_{fouling} \]
Fouling resistance, given in \(\frac{hrf^{2+P}}{BTU}\)
Example - overall heat transfer coefficient 3 resistances (2 convective, 1 conductive)

If you have a hot fluid inside a cylindrical metal pipe with a cold fluid outside, then you can expect to have 3 resistances (2 convective in the fluids and 1 conductive in the solid). The overall heat transfer coefficient can then be written as follows:

\[
R_{\text{total}} = \frac{1}{U_o A_o} = \frac{1}{U_i A_i} = \frac{1}{h_o A_o} + \frac{\Delta r_{pipe}}{k_{pipe} A_{LM,pipe}} + \frac{1}{h_i A_i}
\]

Multiplying through by \(A_o\) or \(A_i\) (reference area) we get:

\[
\frac{1}{U_o} = \frac{1}{h_o} + \frac{A_o \Delta r_{pipe}}{k_{pipe} A_{LM,pipe}} + \frac{A_o}{h_i A_i} \quad \text{and} \quad \frac{1}{U_i} = \frac{A_i}{h_o A_o} + \frac{A_i \Delta r_{pipe}}{k_{pipe} A_{LM,pipe}} + \frac{1}{h_i}
\]

Simplifying with the area definitions (\(A=\pi DL\), \(A_{LM} = \frac{A_1-A_2}{\ln(A_1/A_2)}\)):

\[
\frac{1}{U_o} = \frac{1}{h_o} + \frac{D_o \ln(D_o/D_i)}{2 k_{pipe}} + \frac{D_o}{h_i D_i} \quad \text{and} \quad \frac{1}{U_i} = \frac{D_i}{h_o D_o} + \frac{D_i \ln(D_o/D_i)}{2 k_{pipe}} + \frac{1}{h_i}
\]

Any of these 3 overall equations can be used to find \(\dot{Q}\):

\[
\dot{Q} = \frac{(T_{b,i} - T_{b,o})}{R_{\text{total}}} = U_o A_o (T_{b,i} - T_{b,o}) = U_i A_i (T_{b,i} - T_{b,o})
\]

And the temperature at any intermediate point can be found from that \(\dot{Q}\):

\[
\dot{Q} = h_i A_i (T_{b,i} - T_{w,i}) = \frac{2\pi k L}{\ln(D_o/D_i)} (T_{w,i} - T_{w,o}) = h_o A_o (T_{w,o} - T_{b,o})
\]

Note on temperature subscripts: \(b = \text{bulk}, w = \text{wall}, o = \text{outside}, i = \text{inside}\)

** Be careful and think through all the resistances you have in your system! Double check that you are using the correct \(A_{LM}\) (especially if you have multiple layers of conduction, each with its own \(A_{LM}\), \(A_i\), and \(A_o\), etc.)
Non-Steady State Problems

In steady state, temperature still changes with location, just not with time! For non-steady state problems, the temperature changes with both time and location. You should only use the Biot number and the Fourier number for non-SS calculations.

From Non-Steady State Conduction lecture slides (copied at the end of this review sheet):

Fig. 10.5 \(\rightarrow\) average temperature (without external thermal resistance) of a slab, cylinder, or sphere with constant surface temperature (denoted \(T_s\) or \(T_b\)); variables: \(\text{Fo}\) and \(\bar{\Theta}\)

Fig. 10.7 \(\rightarrow\) average temperature of a slab with external thermal resistance (bulk temperature beyond film denoted \(T_f\)); variables: \(\text{Fo}, \text{Bi}, \text{and} \bar{\Theta}\)

Fig. 10.8 \(\rightarrow\) average temperature of a sphere with external thermal resistance (bulk temperature beyond film denoted \(T_f\)); variables: \(\text{Fo}, \text{Bi}, \text{and} \bar{\Theta}\)

From class handout with plots:

Fig. 11.1.2 \(\rightarrow\) transient radial temperature profiles (at position \(r\)) in a sphere with constant surface temperature (\(\text{Bi}>1\), little/no external resistance); variables: \(r/R, \text{Fo}, \text{and} \bar{\Theta}\)

Fig. 11.1.3 (same as 10.6 above) \(\rightarrow\) average temperature of a slab, cylinder, or sphere with constant surface temperature (\(\text{Bi}>1\), little/no external resistance); variables: \(\text{Fo}\) and \(\bar{\Theta}\)

Fig. b) \(\rightarrow\) transient mid-plane temperature (position at center of thickness) for a slab with external thermal resistance (convective \(h\)); variables: \(\text{Fo}, \text{Bi}, \bar{\Theta}\)

Fig. d) \(\rightarrow\) transient axial temperature (at center-line \(r=0\) position) for a long cylinder with external thermal resistance (convective \(h\)); variables: \(\text{Fo}, \text{Bi}, \bar{\Theta}\)

Fig. f) \(\rightarrow\) transient center temperature (at \(r=0\) position) for a sphere with external thermal resistance (convective \(h\)); variables: \(\text{Fo}, \text{Bi}, \bar{\Theta}\)

Also figure (a) for equation coefficients, and figures (c), (e), and (g) for the ratio of \(\bar{\Theta}\) at the surface to \(\bar{\Theta}\) at the center of the solid slab, cylinder, and sphere, respectively.
Heat Transfer Correlations

For heat transfer by forced convection, \( Nu = f(Re, Pr, \Phi) \) and empirical correlations exist for different geometries to calculate the convective heat transfer coefficient (\( h \)) in the fluid film. For natural convection (density-driven circulation) the notes give a correlation for \( Nu = f(Gr, Pr) \), where \( Gr \) is the Grashof number.

\[
\begin{align*}
Nu &= 2.0 + 0.6Re^{1/2} Pr^{1/3} & \text{Flow past a single sphere (external forced convection)} \\
Nu Pr^{-1/3} &= 0.35 + 0.56Re^{0.52} & \text{Flow normal to tubes (external forced convection)} \\
Nu &= 0.023Re^{0.8} Pr^{1/3} \Phi^{0.14} & \text{Sieder-Tate equation for flow in pipes; use } Re = \frac{\rho D_{eq} V}{\mu} \\
&\text{Note: The Colburn analogy in the notes is a rearranged} \\
&\text{Sieder-Tate equation that gives the } j \text{-factor for heat} \\
&\text{transfer (}j_H\text{) in terms of the Fanning friction factor.} \\
Nu_o &= 0.2Re^{0.6} Pr^{1/3} \Phi^{0.14} & \text{Donohue Correlation for shell side flow; use } Re = \frac{D_o Ge}{\mu} \\
&\text{\( G_e \) is average flow rate } [=\frac{\text{kg}}{m^2s} D_o = \text{OD of 1 tube}}
\end{align*}
\]

Calculating Re in Heat Exchangers

Use these to calculate the Reynolds number used in the Sieder-Tate and Donohue Correlations in order to find the outside heat transfer coefficients (\( h_o \) values)

Countercurrent:

\[
R_H = \frac{S}{L_p} \quad \text{Definition of hydraulic radius} \\
S = \text{cross sectional area}, \quad L_p = \text{wetted perimeter} \\
D_{eq} = 4R_H \quad \text{Definition of equivalent diameter}
\]

Shell-and-Tube:

\[
G_e = \sqrt{G_b G_c}; \quad G_b = \frac{m}{S_b}; \quad G_c = \frac{m}{S_c} \quad \text{Geometric average of horizontal and vertical flow rates}
\]

\[
S_b = f_b \frac{\pi D_b^2}{4} - f_b N \frac{\pi D_o^2}{4} \quad \text{Cross-sectional area available for shell horizontal flow} \\
&\text{\( D_s = \text{shell ID}, \quad D_o = \text{OD of 1 tube, } N = \# \text{ of tubes/shell pass} \)} \\
f_b = \text{fraction of shell cross-section not covered by baffle}
\]

\[
S_c = PD_s \left(1 - \frac{D_o}{p}\right) \quad \text{Cross-sectional area available for shell vertical flow} \\
&\text{\( D_s = \text{shell ID}, \quad D_o = \text{OD of 1 tube, } p = \text{tube pitch,} \)} \\
P = \text{baffle spacing}
\]
Figure 10.5: Average temperatures during unsteady-state heating or cooling of a large slab, and infinitely long cylinder, or a sphere.
Figure 10.7: Change with time of the average temperature of a slab with external convective resistance.

Figure 10.8: Change with time of the average temperature of a sphere with external convective resistance.
1-2 exchangers

**FIGURE 15.3**
2-4 exchangers

FIGURE 15.4
A 2-4 exchanger.