Solution to HW #5

problem 1. The subsets $\{V \cap \Phi_\alpha\}$ covers the whole set V:

$$\bigcup_\alpha \Phi_\alpha = M \longrightarrow \bigcup_\alpha (V \cap \Phi_\alpha) = V \cap \left(\bigcup_\alpha \Phi_\alpha\right) = V \cap M = M$$

Therefore each pair $\{\Phi_\alpha|_{V\cap U_\alpha}, V \cap U_\alpha\}$ defines a local chart for V.

Also if we look at the transition map between two overlapping charts $\{V \cap U_\alpha\}$ and $\{V \cap U_\beta\}$

$\Phi_\beta|_{V\cap U_\beta} \circ \Phi_\alpha^{-1}|_{V\cap U_\alpha}$ is clearly smooth if the map

$\Phi_\beta \circ \Phi_\alpha^{-1}$ is smooth.

problem 2.

Cas I. starting ~~pa~~ set: $\{Z_1Z_2, X_1+X_2\}$

$[Z_1Z_2, X_1+X_2] = 2i(Y_1Z_2 + Z_1Y_2)$ (New)

$[Z_1Z_2, Y_1Z_2+Z_1Y_2] = -2i(X_1+X_2)$ (Not new)

$[X_1+X_2, Y_1Z_2+Z_1Y_2] = 2i(Z_1Z_2 - Y_1Y_2)$ new

$[Z_1Z_2, Y_1Y_2] = 0$

$[X_1+X_2, Y_1Y_2] = 2i(Z_1Y_2+Y_1Z_2)$ (not new)

$\Longrightarrow \{Z_1Z_2, X_1+X_2\}_L = \text{span}\{Z_1Z_2, Y_1Y_2, Y_1Z_2+Z_1Y_2, X_1+X_2\}$

$\Longrightarrow$ This is a 4-dim algebra.

Case II. starting set $\{Z_1Z_2, X_1, X_2\}$

$[Z_1Z_2, X_1] \propto Y_1Z_2$ (new)

$[Z_1Z_2, X_2] \propto Z_1Y_2$ (new)

$[X_1, X_2] = 0$ (not new)

$[Z_1Z_2, Y_1Z_2] \propto X_1$ $\cancel{X_2}$ ~~(new)~~ (not new)

$[Z_1Z_2, Z_1Y_2] \propto X_2$ (not new)

$[X_1, Y_1Z_2] \propto Z_1Z_2$ (not new)

$[X_2, Y_1Z_2] \propto Y_1Y_2$ (new)

$[X_1, Y_2Z_1] \propto Y_1Y_2$ (not new)

$[X_2, Y_2Z_1] \propto Z_1Z_2$ (not new)

$[Z_1Z_2, Y_1Y_2] = 0$ (not new)

$[X_1, Y_1Y_2] \propto Z_1Y_2$ (not new)

$[X_2, Y_1Y_2] \propto Y_1Z_2$ (not new)

$$\Longrightarrow \{Z_1Z_2, X_1, X_2\}_{\mathcal{L}} = \operatorname{span}\{Z_1Z_2, X_1, X_2, Y_1Z_2, Z_1Y_2, Y_1Y_2\}$$

dim = 6.