HW 3, Solution

The task of identifying an unknown quantum state is called quantum state tomography (QST).

Similar to system identification problem in classical systems, we need to complete a number of measurements on a quantum system to reveal its state.

Problem: How to identify a density matrix of a quantum system with Hilbert space of dimension $\underline{\underline{N}}$.

$$\boxed{\dim(\mathcal{H}_S) = N}$$

The expectation value of an observable $\hat{O}$ is $Tr(\hat{\rho}\hat{O})$ that can be achieved by measuring $\hat{O}$

on an large enough ensemble of the same systems all in state ρ.

ρ is a trace one $N \times N$ positive matrix so it has

N^2-1 unknown parameters (elements).

Therefor if we measure N^2-1 linearly independent observable $\{\hat{O}_1, \dots, \hat{O}_{N^2-1}\}$ then we will have enough number of equations to solve for ρ.

$$
\begin{cases}
P_1 = \mathrm{Tr}(\hat{O}_1 \rho) \\
P_2 = \mathrm{Tr}(\hat{O}_2 \rho) \\
\vdots \\
P_{N^2-1} = \mathrm{Tr}(\hat{O}_{N^2-1} \rho)
\end{cases}
$$

There for ρ is simply to solution of this system of linear equations.