HW #2 - Solutions

1) Zero potential $V(x)=0$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$$

Looking for special solution $\psi(x,t) = f(t)\,\phi(x)$

$$i\hbar\,\dot{f}(t)\,\phi(x) = -\frac{\hbar^2}{2m} f(t)\frac{d^2\phi}{dx^2}$$

$$\Rightarrow \frac{\dot{f}(t)}{f(t)} = \frac{i\hbar}{2m}\frac{d^2\phi/dx^2}{\phi}$$

Assume this ratio is equal to $\boxed{\hbar i\omega}$.

$$\frac{\dot{f}(t)}{f(t)} = i\omega \Longrightarrow f(t) = A e^{-i\omega t}$$ for some constant A.

$$\frac{i\hbar}{2m}\frac{d^2\phi/dx^2}{\phi} = -i\omega \Longrightarrow \frac{d^2\phi}{dx^2} = -\frac{\omega(2m)}{\hbar}\phi, \quad \boxed{\frac{2\omega m}{\hbar} = +k^2}$$

$$\Longrightarrow \phi(x) = B e^{ikx} \text{ or } B e^{-ikx}$$

Therefore an special solution will be

$$\psi_{\omega}(x) = C_{+} e^{i(kx-\omega t)} + C_{-} e^{i(-kx-\omega t]}$$

with $k = \sqrt{\frac{2m\omega}{\hbar}}$

Therefore the general solution will be a superposition of these special solutions.

$$\psi(x,t) = \int C_{+}(\omega)\, e^{i(kx-\omega t)}\, d\omega + \int C_{-}(\omega)\, e^{-i(kx+\omega t)}\, d\omega$$

Normalization Condition: $\int |\psi(x,t)|^{2} dx = 1$

$$\Rightarrow \int d\omega \left(|C_{+}(\omega)|^{2} + |C_{-}(\omega)|^{2} \right) = 1$$ ✓

- For the case of a mono chromatic wave function C_+ & C_- ~~are~~ must be delta functions which is not a real physical function.

2) state of the system and probe after time $\underline{t}$ is

$$\rho_{sp}(t) = e^{-iH_{sp}t} \rho_{sp}(0) e^{iH_{sp}t}$$

1- The probability of the k'th outcome is

$$\text{prob}(k) = \text{Tr}\Big(I_s \otimes P_k \, \rho_{sp}(t)\Big) = \text{Tr}\Big(P_k \, \text{Tr}_s(\rho_{sp}(t))\Big)$$

System identity operator

2- The collapsed state of the system & probe is

$$\rho_{sp}^{(k)} = \frac{(I\otimes P_k)\, \rho_{sp}(t)\, (I\otimes P_k)}{\text{Tr}(I\otimes P_k \, \rho_{sp}(t))}$$

$$= \frac{(I\otimes P_k)\, e^{-iH_{sp}t} \rho_s\otimes\rho_p \, e^{iH_{sp}t}\, (I\otimes P_k)}{\text{prob.}(k)}$$