Feedback Cooling a nano-mechanical resonator (NEM

NEMS (systems) are one of the main paradigms for realization of quantum feedback protocols.

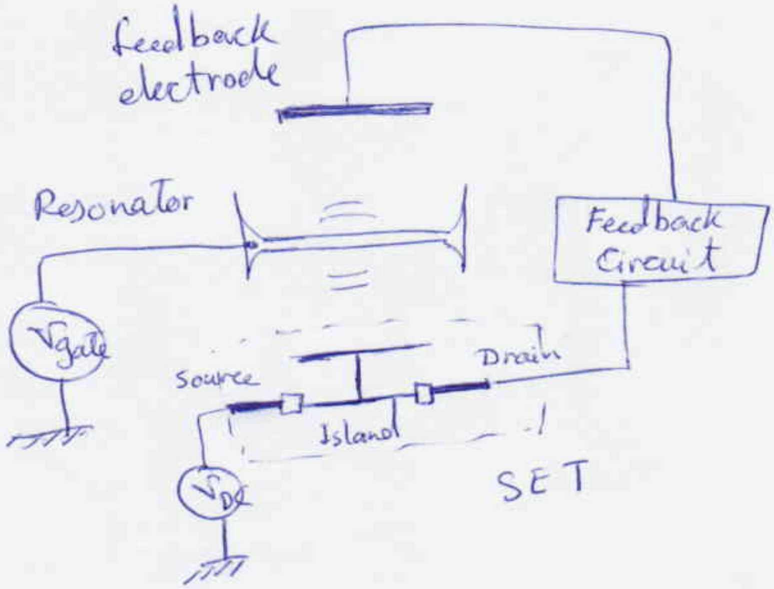
References: 1- A. Hopkins et al., PRB 68, 235328 (2003).

2- A. Naik et al., Nature 443, 193 (2006).

NEMRs are typically built with few-μm length and resonance frequency of several hundreds (tens) of MHz (Radio frequency). The resonator ~~implemented~~ Fabricated in the second reference above is a SiN-Al NEMR with resonance frequency 21.9 MHz.

Control Goal: suppressing the vibrations of a NEMR

set up as given in Ref 1:



~~A~~ The resonator biased with the voltage V_{gate} is coupled to a single electron transistor and an electrode that ~~applies~~ applies a feedback force.

The SET acts as the probe for continuous measurement.

Measurement principle: NEMR and SET form a capacitor. Therefore vibrations of the resonator vary the charge on the transister island that can be detected in the current of the SET.

Feedback controler: The top electrode ~~applies~~ creates some voltage & therefore applies force on the NEMR to kill its vibrations.

The Hamiltonian of the NEMR is described by

$$H = \frac{P^2}{2m} + \frac{1}{2} m \omega_0^2 X^2$$

where P and X are momentum & position operators.

m: mass of the resonator

ω_0: natural frequency

The transistor is measuring the position of the resonator and therefore the observable X.

The stochastic master equation describing both the continuous measurement and decoherence processes:

$$d\rho = -\frac{i}{\hbar}[H,\rho]dt - \frac{i\Gamma}{2\hbar}[x,\{p,\rho\}_+]dt$$
$$-\left(k+\beta+\frac{m\omega_0\Gamma}{2\hbar}\coth\left(\frac{\hbar\omega_0}{2k_BT}\right)\right)[X,[X,\rho]]dt$$
$$+\sqrt{2k}\,(X\rho+\rho X-2\langle X\rangle\rho)\,dW$$

and the measurement record is

$$dy = \langle x\rangle dt + \frac{1}{\sqrt{8k}}dW$$

k: measurement strength

$-\beta[X,[X,\rho]]$: excess dephasing noise

Γ : Terms with coefficient represent the thermal noise

Feedback force:

Consider a linear force: $F = -\gamma \left(m\omega_0 \langle x \rangle + \langle p \rangle \right)$

force strength

In the presence of this feedback the system Hamiltonian becomes

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega_0^2 x^2 - \gamma \left(m \omega_0 \langle x \rangle + \langle p \rangle \right) x$$

Solution: If the initial density matrix is Gaussian in X and P, it remains Gaussian at all times. Therefore it is enough to look at the evolution of means and variances to fully describe the evolution of the system.

Means:

$$d\langle x\rangle = \frac{\langle p\rangle}{m} + 2\sqrt{2k}\,\sigma_x^2\, dW$$

$$d\langle p\rangle = -m\omega^2\langle x\rangle\, dt - \Gamma\langle p\rangle\, dt - \gamma(m\omega\langle x\rangle + \langle p\rangle)\, dt$$

$$+ 2\sqrt{2k}\,\sigma_{xp}^2\, dW$$

Variances:

$$\dot{\sigma}_x^2 = \frac{2}{m}\sigma_{xp}^2 - 8k(\sigma_x^2)^2$$

$$\dot{\sigma}_p^2 = -2m\omega^2\sigma_{xp}^2 - 8k(\sigma_{xp}^2)^2 + 2\hbar^2 k$$

$$+ 2\hbar^2\left[\beta + \frac{m\omega_c\Gamma}{2\hbar}\coth\frac{\hbar\omega_c}{2k_BT}\right]$$

$$\dot{\sigma}_{xp}^2 = \frac{\sigma_p^2}{m} - m\omega^2\sigma_x^2 - \frac{\Gamma}{2}\sigma_{xp}^2 - 8k\,\sigma_x^2\sigma_{xp}^2$$

where

$$\sigma_{xp}^2 = \frac{1}{2}\langle xp + px\rangle - \langle x\rangle\langle p\rangle .$$

Under the assumption of strong measurement $\gamma \gg \Gamma$ the steady-state solutions are

$$
\begin{cases}
\sigma_x^2 = \dfrac{\sqrt{2}\,\omega}{8\kappa}\sqrt{\Lambda} \\[2ex]
\sigma_p^2 = \dfrac{\sqrt{2}\,m^2\omega^3}{8\kappa}\left[\sqrt{\Lambda} + \Lambda^{3/2}\right] \\[2ex]
\sigma_{xp}^2 = \dfrac{m\omega^2}{8\kappa}\,\Lambda
\end{cases}
$$

with $\Lambda = \left[1 + 16\dfrac{\kappa\hbar^2}{m^2\omega^4}\left\{\kappa + \beta + \dfrac{m\omega_0\Gamma}{2\hbar}\coth\dfrac{\hbar\omega}{2kT}\right\}\right]^{1/2} - 1$

The average energy assuming $\gamma \gg \omega$

$$E = \frac{m\omega^3}{8\kappa}\left[\sqrt{2\Lambda} + \Lambda + \frac{\sqrt{2}}{2}\Lambda^{3/2} + \frac{\omega}{4\gamma}\Lambda^2\right]$$

Control objective: choose the feedback strength γ such that the energy is minimum.

System parameters as described in Ref. 1

$T = 100\,mK$, $\omega_0 = 12\,MHz$,

Geometric sizes: $6\,\mu m \times 50\,nm \times 150\,nm$

$m = 10^{-16}\,kg$, $\beta = 10^{31}\,m^{-2}s^{-1}$, $k = 0.184\,\beta$

~~the~~ $\Gamma = 0.3\,MHz$.

With this parameters the minimum achievable temperature is around $2\,mK$.

$$\left(T_{eff} = \frac{E_{min}}{k_B} \right)$$