EE290O
Homework 7: Generalized Measurement

Consider the implementation of a generalized measurement process as described in the class. The probe and system are both infinite dimensional and their Hilbert space basis are given by the their position eigenstates, $|x\rangle$ for the system and $|y\rangle$ for the probe. The total evolution of the system and probe is a unitary G defined as below

$$G|x\rangle|y\rangle = |x\rangle|x+y\rangle \tag{1}$$

Problem 1: Show that the transformation G is unitary.

Now assume the probe is initially in the pure state

$$|\theta\rangle = \int_{\infty}^{\infty} d\xi [\frac{1}{\sqrt{2\pi\Delta^2}} \exp(\frac{-\xi^2}{2\Delta^2})]^{\frac{1}{2}} |\xi\rangle \tag{2}$$

and after applying unitary G the probe is measured in the momentum basis $\{|p\rangle\}$, that means the momentum of the probe is measured via a strong measurement with projection operators $\{|p\rangle\langle p|\}$.
problem 2: Show that the generalized measurement operator corresponding to the measurement result p is

$$M_p = (\frac{2\Delta^2}{\pi})^{1/4} \exp(p^2\Delta^2) \exp(-ipX) \tag{3}$$

with the position operator X. Remember the inner product $\langle p|x\rangle = (2\pi)^{-1/2} e^{-ixp}$.

Reference: H. M. Wiseman, G. J. Milburn, Quantum Measurement and Control, Cambridge University Press (2009).