EE290O
Homework 6: Coherent Control

Problem 1: (Two Spin/Qubit control) In class we considered the controllability of two interacting spins via local magnetic fields. Here we are going to examine the controllability criteria with complete global control The corresponding Hamiltonian is

$$H = \omega_0 Z_1 \otimes Z_2 + \gamma B_x(t)(X_1 + X_2) + \gamma B_y(t)(Y_1 + Y_2) + \gamma B_z(t)(Z_1 + Z_2) \quad (1)$$

with control parameters $\{B_x, B_y, B_z\}$. What can you say about the controllability of this system?
How is the following control system different from the one above?

$$H = \omega_0 Z_1 \otimes Z_2 + \gamma B_x(t)(X_1 + X_2) + \gamma B_z(t)(Z_1 + Z_2) \quad (2)$$

Problem 2: (Algebra)

I) Give a basis for $su(n)$ and one for $sp(\frac{n}{2})$.

II) Find a transformation $X \in Sp(2)$ which performs the state transfer

$$\psi_1 = X\psi_0 \quad (3)$$

where $\psi_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\psi_0 = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix}$.

Reference: Domenico D'Alessandro, Introduction to Quantum Control and Dynamics, Chapman & Hall (2008).