

Matlab Programming in 240: The Ising Model

The goal of the programming for this semester is to write a program so that we can explore the behavior of the 2-d Ising model using Monte Carlo simulations. We will be using Matlab to write M-files that will help us look at how changes to the system (lattice size, temperature, magnetic field) affect its properties (magnetization).

Here's a little background on the Ising model and the motivation behind studying it: the Ising model was named after physicist Ernst Ising and is one of the most widely studied models in statistical mechanics. In his 1925 PhD thesis, he correctly showed that for the 1-D model, there is no phase transition. He went on to state that this model not exhibit a phase transition in any dimension. Unfortunately for him, this was shown to be incorrect when the 2-D Ising model was solved analytically (in the absence of a magnetic field) in 1944 by Lars Onsager. A motivating factor behind its study is its wide applicability. For instance, the grand canonical ensemble (Ξ) for a lattice gas model can be mapped onto the canonical ensemble (Q) of the Ising model. Furthermore, it can be used to look at ferromagnetism and anti-ferromagnetism. More information can be found in Chapter 5 of Chandler (sections 5.1 and 5.2). You should also considering visiting the sample Ising models that are linked from the course homepage to get a qualitative feel for the features of these models.

In order to construct such a system we will need to become familiar with a variety of techniques including random number generators, periodic boundary conditions, Monte Carlo simulations, the Metropolis algorithm (the heart of the Ising model) and (time permitting) correlation functions. Following is an outline of the sequence leading up to a completed Ising model.

Ising Model Outline (tentative)

- 1) Generate a sequence of random numbers and calculate their mean and standard deviation. Look at correlations among different elements in the lattice.
- 2) Create a square lattice and populate it with 1s. Pick different lattice elements and flip the sign and report how the mean magnetization changes with "time."
- 3) Use periodic boundary conditions. Pick a lattice element and swap its value with a nearest neighbor, treating the boundaries properly.
- 4) Randomly populate a lattice with +1 and -1. Compute the number of AA, BB, and AB nearest neighbors. Assign values for the contact energies and calculate E , Q , A/N , and G/NRT . Change the value of T and the contact energies.
- 5) Ising model. Include a probability for a lattice element to be flipped based on the value of the temperature and energy of interactions with neighboring lattice elements. Study how the magnetization changes with "time."
- 6) Include the effect of an external magnetic field, H , on the magnetization.