Table of Dimensionless Numbers

Group	Definition	Interpretation
Archimedes Number (Ar)	$\frac{g\rho L^3}{\mu^2}$	Ratio of gravitational force to viscous force
Arrhenius Number (α)	$\frac{E_o}{RT}$	Ratio of activation energy to thermal energy
Bingham Number (Bm)	$\frac{\tau_y L}{\mu V}$	Ratio of yield stress to viscous stress
Biot Number (Bi)	$\frac{hL}{k}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance
Mass Transfer Biot Number (Bi_m)	$\frac{h_m L}{k}$	Ratio of the internal species transfer resistance to the boundary layer species transfer resistance
Blake Number (Bl)	$\frac{V\rho}{\mu(1-\varepsilon)s}$	Ratio of inertial force to viscous force in flow through beds of solids
Bodenstein Number (Bs)	$\frac{vL}{D_{v,a}}$	Mass transfer number used in reactor calculations with velocity, length, and axial diffusivity
Bond Number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces
Brinkman Number (Br)	$\frac{\mu v^2}{k(T-T_o)}$	Ratio of viscous dissipation to thermal conduction
Capillary Number (Ca)	$\frac{\mu V}{\sigma}$	Ratio of viscous force to surface tension force
Cauchy Number (C)	$\frac{\rho v^2}{E}$	Ratio of inertial force to compressibility force (modulus)
Cavitation Number (σ_C)	$\frac{2(p-p_v)}{\rho v^2}$	Ratio of excess local static pressure head to velocity head
Coefficient of Frication (C_f)	$\frac{\tau}{\rho v^2/2}$	Dimensionless surface shear stress
Condensation Number (Co)	$\frac{g\rho^2 \Delta H_{vap} L^3}{k\mu\Delta T}$	Used in condensation calculations
Dean Number (De)	$\mathrm{Re}\left(\frac{L}{2R}\right)^{\frac{1}{2}}$	Reynolds number times the centrifugal force over the inertial force
Drag Coefficient (C_d)	$\frac{g(\rho-\rho_f)L}{\rho v^2}$	Ratio of gravitational force to inertial force used in free settling velocities and resistance to flow

Ha Hatta Number Da, Φ Ga_ω - diffusional Grashof #

Eckert Number (Ec)	$\frac{v^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference
Elasticity Number (El)	$\frac{\theta\mu}{\rho R^2}$	Ratio of elastic force to inertial force in viscoelastic flow
Etovos Number (Eo)	$\frac{(\rho - \rho_f)L^2}{\sigma}$	Ratio of gravitational force to surface force; equivalent to Bo
Euler Number (Eu)	$\frac{\Delta p}{\rho v^2}$	Ratio of pressure force to inertial force
Fourier Number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time.
Mass Transfer Fourier Number (Fo_m)	$\frac{Dt}{L^2}$	Ratio of the species diffusion rate to the rate of species storage. Dimensionless time.
Friction Factor (f)	$\frac{\Delta p}{(L/D)(\rho v^2/2)}$	Dimensionless pressure drop for internal flow
Froude Number (Fr)	$\frac{v^2}{gL}$	Ratio of inertial force to gravitational force
Galileo Number (Ga)	$\frac{gD^3\rho^2}{\mu^2}$	Reynolds number times the gravitational force over the viscous force
Gratz Number (Gz)	$\frac{\dot{m}c_p}{kL}$	Ratio of the thermal capacity to convective heat transfer
Grashof Number (Gr)	$\frac{g\beta(T_s - T_\infty)L^3}{v^2}$	Ratio of buoyancy to viscous forces
Colburn j factor (j_H)	$St\,\mathrm{Pr}^{\frac{2}{3}}$	Dimensionless heat transfer coefficient
Colburn j factor (j_m)	$St_m Sc^{\frac{2}{3}}$	Dimensionless mass transfer coefficient
Hodgson Number (H)	$\frac{fV\Delta p}{Qp}$	Time constant of the system over the period of pulsation (f is frequency, Q is volumetric flow rate)
Jakob Number (Ja)	$\frac{c_p(T_s - T_{sat})}{\Delta H_{vap}}$	Ratio of sensible to latent energy absorbed during liquid-vapor phase change
Knudsen Number (Kn)	$\frac{\lambda}{L}$	Length of mean free path relative to characteristic length of system
Lewis Number	$\frac{\alpha}{D}$	Ratio of thermal and mass diffusivities
Mach Number (M)	$\frac{v}{v_{sonic}}$	Velocity divided by the speed of sound

Nusselt Number (Nu)	$\frac{hL}{k}$	Dimensionless temperature gradient at the surface
Ohnesorge Number (Z)	$\frac{\mu}{\sqrt{\rho L \sigma}}$	Ratio of the viscous force to the square root of the product of inertial and surface forces
Peclet Number (Pe)	$\frac{vL}{\alpha}$	Dimensionless independent heat transfer parameter
Mass Transfer Peclet Number (Pe_m)	$\frac{vL}{D}$	Dimensionless independent mass transfer parameter
Pipeline Parameter (ρ^n)	$\frac{av_o}{2gh}$	Maximum water hammer pressure rise over twice the static pressure (h is the static head, a is the wave velocity)
Power Number (N_p)	$\frac{P}{N^3 \rho L^5}$	Ratio of drag force to inertial force for power consumption calculations (N is rate of rotation, P is power)
Prandtl Number (Pr)	$\frac{v}{\alpha}$	Ratio of the momentum and thermal diffusivities
Rayleigh Number (Ra)	$Gr\,\mathrm{Pr}$	Used in heat transfer and free convection calculations
Reynolds Number (Re)	$\frac{vL\rho}{\mu}$	Ratio of the inertial and viscous forces
Schmidt Number (Sc)	$\frac{v}{D}$	Ratio of the momentum and mass diffusivities
Sherwood Number (Sh)	$\frac{h_m L}{D}$	Dimensionless concentration gradient at the surface
Stanton Number (St)	$\frac{h}{\rho v c_p}$	Modified Nusselt number
Mass Transfer Stanton Number (St_m)	$\frac{h_m}{v}$	Modified Sherwood number
Strouhal Number (Sr)	$\frac{fL}{v}$	Proportional to reciprocal of vortex spacing (f is frequency)
Weber Number (We)	$\frac{\rho v^2 L}{\sigma}$	Ratio of inertia to surface tension forces

es of Re and
ecall that the
it flow. In any
duce turbulent
large relative
flow is main-
sively less im-
amplified to a
gnitude of the
ess δ. With in-
ces to become
scosity do not
s.
rom its defini-
usivity α. The
of momentum
undary layers.
f gases is near
n are compara-
or greatly
ls e A.5).
e value of Pr
rmal boundary
by diffusion is
that
(6.62)
d metal $\delta_t \gg \delta$
n 6.47, provides
ransport by dif-
tively. For con-
es the relative
(6.63)
umber (Le). It is
(6.64)
d mass transfer
(6.65)

The Lewis number is therefore a measure of the relative thermal and concentration boundary layer thicknesses. For most applications it is reasonable to assume a value of $n = 1/3$ in Equations 6.62, 6.63, and 6.65.

Table 6.2 lists the dimensionless groups that appear frequently in heat and mass transfer. The list includes groups already considered, as well as those yet

TABLE 6.2 Selected dimensionless groups of heat and mass transfer

Group	Definition	Interpretation
Biot number (Bi)	$\frac{hL}{k_s}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance.
Mass transfer Biot number (Bi_m)	$\frac{h_m L}{D_{AB}}$	Ratio of the internal species transfer resistance to the boundary layer species transfer resistance.
Bond number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces.
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress.
Eckert number (Ec)	$\frac{V^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference.
Fourier number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time.
Mass transfer Fourier number (Fo_m)	$\frac{D_{AB}t}{L^2}$	Ratio of the species diffusion rate to the rate of species storage. Dimensionless time.
Friction factor (f)	$\frac{\Delta p}{(L/D)(\rho u_m^2/2)}$	Dimensionless pressure drop for internal flow.
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	Ratio of buoyancy to viscous forces.
Colburn j factor (j_H)	$St\, Pr^{2/3}$	Dimensionless heat transfer coefficient.
Colburn j factor (j_m)	$St_m\, Sc^{2/3}$	Dimensionless mass transfer coefficient.
Jakob number (Ja)	$\frac{c_p(T_s - T_{sat})}{h_{fg}}$	Ratio of sensible to latent energy absorbed during liquid–vapor phase change.
Lewis number (Le)	$\frac{\alpha}{D_{AB}}$	Ratio of the thermal and mass diffusivities.
Nusselt number (Nu_L)	$\frac{hL}{k_f}$	Dimensionless temperature gradient at the surface.
Peclet number (Pe_L)	$\frac{VL}{\alpha} = Re_L Pr$	Dimensionless independent heat transfer parameter.
Prandtl number (Pr)	$\frac{c_p \mu}{k} = \frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities.
Reynolds number (Re_L)	$\frac{VL}{\nu}$	Ratio of the inertia and viscous forces.
Schmidt number (Sc)	$\frac{\nu}{D_{AB}}$	Ratio of the momentum and mass diffusivities.
Br Brinkman #	$\frac{\mu V_b^2}{k(T_b - T_0)}$	Ratio of viscous dissipation to thermal conduction

3

TABLE 6.2 *Continued*

Group	Definition	Interpretation
Sherwood number (Sh_L)	$\frac{h_m L}{D_{AB}}$	Dimensionless concentration gradient at the surface.
Stanton number (St)	$\frac{h}{\rho V c_p} = \frac{Nu_L}{Re_L\, Pr}$	Modified Nusselt number.
Mass transfer Stanton number (St_m)	$\frac{h_m}{V} = \frac{Sh_L}{Re_L Sc}$	Modified Sherwood number.
Weber number (We)	$\frac{\rho V^2 L}{\sigma}$	Ratio of inertia to surface tension forces.

to be introduced for special conditions. As a new group is confronted, its definition and interpretation should be committed to memory. Note that the *Grashof number* provides a measure of the ratio of buoyancy forces to viscous forces in the velocity boundary layer. Its role in free convection (Chapter 9) is much the same as that of the Reynolds number in forced convection. The *Eckert number* provides a measure of the kinetic energy of the flow relative to the enthalpy difference across the thermal boundary layer. It plays an important role in high-speed flows for which viscous dissipation is significant. Note also that, although similar in form, the Nusselt and Biot numbers differ in both definition and interpretation. Whereas the Nusselt number is defined in terms of the thermal conductivity of the fluid, the Biot number is based on the solid thermal conductivity, Equation 5.9.

6.7 *Boundary Layer Analogies*

As engineers, our interest in boundary layer behavior is directed principally toward the dimensionless parameters C_f, Nu, and Sh. From knowledge of these parameters, we may compute the wall shear stress and the convection heat and mass transfer rates. It is therefore understandable that expressions that relate C_f, Nu, and Sh to each other can be useful tools in convection analysis. Such expressions are available in the form of *boundary layer analogies.*

6.7.1 The Heat and Mass Transfer Analogy

If two or more processes are governed by dimensionless equations of the same form, the processes are said to be *analogous*. Clearly then, from Equations 6.40 and 6.41 and the boundary conditions, Equations 6.43 and 6.44, of Table 6.1, convection heat and mass transfer are analogous. Each of the differential