21.17 The diffusion coefficient of glucose is $5.7 \times 10^{-10}\ \mathrm{m^2\,s^{-1}}$. Calculate the time required for a glucose molecule to diffuse through **(a)** 10,000 Å and **(b)** 0.10 m.

The root-mean-square distance is related to time via

$$\sqrt{\overline{x^2}} = \sqrt{2Dt}$$

Thus,

$$t = \frac{\overline{x^2}}{2D}$$

(a)

$$t = \frac{\left(10,000 \times 10^{-10}\ \mathrm{m}\right)^2}{(2)\left(5.7 \times 10^{-10}\ \mathrm{m^2\,s^{-1}}\right)} = 8.8 \times 10^{-4}\ \mathrm{s}$$

(b)

$$t = \frac{(0.10\ \mathrm{m})^2}{(2)\left(5.7 \times 10^{-10}\ \mathrm{m^2\,s^{-1}}\right)} = 8.8 \times 10^{6}\ \mathrm{s} = 100\ \mathrm{days}$$

21.18 The diffusion coefficient of sucrose in water at 298 K is $0.46 \times 10^{-5}\ \mathrm{cm^2\,s^{-1}}$, and the viscosity of water at the same temperature is $0.0010\ \mathrm{N\,s\,m^{-2}}$. From these data, estimate the effective radius of a sucrose molecule.

Rearrange Equation 21.25 to give

$$r = \frac{k_B T}{6\pi\eta D}$$

$$= \frac{\left(1.381 \times 10^{-23}\ \mathrm{J\,K^{-1}}\right)(298\ \mathrm{K})}{6\pi\left(0.0010\ \mathrm{N\,s\,m^{-2}}\right)\left(0.46 \times 10^{-9}\ \mathrm{m^2\,s^{-1}}\right)}$$

$$= 4.7 \times 10^{-10}\ \mathrm{m} = 4.7\ \text{Å}$$

21.19 From the diffusion coefficients listed in Table 21.3, estimate the radius and molecular volume of myoglobin and hemoglobin. What conclusion can you draw from the results?

The effective radius of a molecule can be calculated by rearranging Equation 21.25. Once the radius is determined, the volume can be calculated assuming the molecule is a sphere.

For myoglobin,

$$r = \frac{k_B T}{6\pi \eta D}$$

$$= \frac{(1.381 \times 10^{-23}\ \mathrm{J\,K^{-1}})\,(298\ \mathrm{K})}{6\pi\,(0.00101\ \mathrm{N\,s\,m^{-2}})\,(0.113 \times 10^{-9}\ \mathrm{m^2\,s^{-1}})}$$

$$= 1.913 \times 10^{-9}\ \mathrm{m} = 19.1\ \text{Å}$$

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\,(1.913 \times 10^{-9}\ \mathrm{m})^3$$

$$= 2.932 \times 10^{-26}\ \mathrm{m^3} = 2.93 \times 10^4\ \text{Å}^3$$

For hemoglobin,

$$r = \frac{k_B T}{6\pi \eta D}$$

$$= \frac{(1.381 \times 10^{-23}\ \mathrm{J\,K^{-1}})\,(298\ \mathrm{K})}{6\pi\,(0.00101\ \mathrm{N\,s\,m^{-2}})\,(0.069 \times 10^{-9}\ \mathrm{m^2\,s^{-1}})}$$

$$= 3.13 \times 10^{-9}\ \mathrm{m} = 31\ \text{Å}$$

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\,(3.13 \times 10^{-9}\ \mathrm{m})^3$$

$$= 1.28 \times 10^{-25}\ \mathrm{m^3} = 1.3 \times 10^5\ \text{Å}^3$$

The volume of hemoglobin is $\frac{1.28 \times 10^{-25}\ \mathrm{m^3}}{2.932 \times 10^{-26}\ \mathrm{m^3}} = 4.4$ times greater than that of myoglobin. Since hemoglobin is a tetramer of myoglobin, this result makes sense.

21.20 Diffusion coefficients have been measured for many solid systems. If the diffusion coefficient of bismuth in lead is $1.1 \times 10^{-16}\ \mathrm{cm^2\,s^{-1}}$ at 20°C, calculate how long it will take (in years) for a bismuth atom to travel 1.0 cm.

Rearrange Equation 21.22 to give

$$t = \frac{\overline{x^2}}{2D}$$

$$t = \frac{\left(1.0 \times 10^{-2}\ \text{m}\right)^2}{(2)\left(1.1 \times 10^{-20}\ \text{m}^2\,\text{s}^{-1}\right)}$$

$$= \left(4.55 \times 10^{15}\ \text{s}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24\ \text{hr}}\right)\left(\frac{1\ \text{year}}{365\ \text{days}}\right)$$

$$= 1.4 \times 10^{8}\ \text{years}$$

21.21 What is the diffusion coefficient of a membrane-bound protein of molar mass 80,000 daltons at 37°C if the viscosity of the membrane is 1 poise ($0.10\ \text{N s m}^{-2}$)? What is the average distance traveled by this protein in 1.0 s? Assume that this protein is an unhydrated, rigid sphere that has a density of $1.4\ \text{g cm}^{-3}$.

Use the volume of 1 protein molecule,

$$V = \left(80,000\ \text{g mol}^{-1}\right)\left(\frac{1\ \text{mol}}{6.022 \times 10^{23}}\right)\frac{1}{\left(1.4\ \text{g cm}^{-3}\right)} = 9.49 \times 10^{-20}\ \text{cm}^3$$

to find the effective radius of the protein

$$V = \frac{4}{3}\pi r^3 = 9.49 \times 10^{-20}\ \text{cm}^3$$

$$r = \left[\frac{(3)\left(9.49 \times 10^{-20}\ \text{cm}^3\right)}{4\pi}\right]^{1/3} = 2.83 \times 10^{-7}\ \text{cm}$$

The diffusion coefficient can be calculated using Equation 21.25.

$$D = \frac{k_B T}{6\pi \eta r}$$

$$= \frac{\left(1.381 \times 10^{-23}\ \text{J K}^{-1}\right)(310\ \text{K})}{6\pi\left(0.10\ \text{N s m}^{-2}\right)\left(2.83 \times 10^{-9}\ \text{m}\right)}$$

$$= 8.03 \times 10^{-13}\ \text{m}^2\,\text{s}^{-1} = 8.0 \times 10^{-13}\ \text{m}^2\,\text{s}^{-1}$$

The average distance traveled by this protein in 1.0 s is

$$\sqrt{\overline{x^2}} = \sqrt{2Dt}$$

$$= \sqrt{(2)\left(8.03 \times 10^{-13}\ \text{m}^2\,\text{s}^{-1}\right)(1.0\ \text{s})}$$

$$= 1.3 \times 10^{-6}\ \text{m} = 1.3 \times 10^{4}\ \text{Å}$$

21.30 The diffusion coefficient of oxygen in air is $0.20\ \text{cm}^2\ \text{s}^{-1}$; the diffusion coefficient of the same gas in water is about 10^4 times smaller. **(a)** Explain the huge difference in the magnitude of these diffusion coefficients. **(b)** Most animal cells are bathed in fluids, so that a hemoglobinlike molecule and a circulatory system are necessary for the purpose of transporting O_2 to their cells and carrying CO_2 away. (The diffusion coefficients of CO_2 in air and in water are comparable in magnitude to those of oxygen.) Because plants do not have a circulatory system, explain how the O_2 and CO_2 gases are transported efficiently in these systems. **(c)** Insects do possess a circulating system but lack a hemoglobinlike molecule. Considering the diffusion coefficients of CO_2 and O_2 in water, do you think it likely that ants, bees, and cockroaches can grow to human size, as they sometimes do in horror movies?

(a) Since diffusion involves molecular motion, it will depend greatly upon the medium through which the molecules in question must move.

(b) Plants have conspicuous intercellular air spaces by which they can take advantage of the large diffusion coefficients of O_2 and CO_2 in the gas phase.

(c) Without a means of efficient transport for O_2 and CO_2, insects are limited to small sizes so that their metabolic processes can obtain enough O_2 via diffusion through their circulating system. Because of this limitation, they can not grow to horror movie size.