Review of Classical Dynamics

1. Classical Turning Points

\[ \frac{dV}{dx} \]

Force on particle: \(- \frac{dV}{dx} \)

Or multivariable potential:

\[ F = - \nabla V \]

2. Deterministic

\[ x(0), \dot{x}(0) \]

Trajectory is completely known classically given initial condition

3. A charged particle \((q, e^-)\) in a Coulombic potential (proton at the origin)

\[ r = \sqrt{x^2 + y^2 + z^2} \]

\[ V(r) = \frac{qQ}{r} \]
\[ \frac{de}{dt} = 0 \quad \Rightarrow \quad p_x = -\frac{\partial V}{\partial x}, \quad p_y = -\frac{\partial V}{\partial y}, \quad p_z = -\frac{\partial V}{\partial z} \]

\[ F_x = -\frac{\partial V}{\partial x} = \frac{2}{\omega} V \left( \sqrt{x^2 + y^2 + z^2} \right) \]

\[ = \frac{-\omega}{r} \]

\[ V = \frac{Q}{r} \]

\[ F_y = \frac{Q}{r^3} \quad F_z = \frac{2Q}{r^3} \]

\[ \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} = -\nabla V(r) = \frac{Q}{r^2} \left( \frac{x \hat{i} + y \hat{j} + z \hat{k}}{r} \right) \]

\[ = \frac{Q}{r^2} \left( \frac{\vec{r}}{r} \right) = \frac{Q}{r^2} \hat{r} \]

\[ \hat{r} \text{ is unit vector in direction } r \]

According to this potential, electrons will always want to minimize energy, thus spiral into protons.

\[ \Rightarrow \]

i) Atoms have no size
ii) Radiate heat

Classically
Experiments that lead to Quantum Mechanics

1) Blackbody Radiation

- An ideal body that emits and absorbs all frequencies.

Rayleigh-Jeans explained classically by using a vibrating electron, and changes in vibration frequency to correlate to spectrum changes.

Rayleigh-Jeans

\[ d\rho(v, T) = \frac{8\pi k_0 T}{c^3} v^2 dv \]  

- Assumes all frequencies allowed for emission.

The theory needs improvement.

Planck - Maybe all frequencies are not allowed.

Let \( E = nh\nu \), where \( n \) is an integer and \( h \) is Planck's constant.

Using this assumption

\[ d\rho(v, T) = \frac{8\pi h}{c^3} \frac{v^2 dv}{e^{hv/k_0 T} - 1} \]

2. Emission Spectrum of H Atoms

\[ \lambda = \frac{1}{n_1^2} - \frac{1}{n_2^2} \text{ cm}^{-1} \]

Rydberg proposed this formula to express calculate the positions of emission lines.

For Lyman: 1, 2, 3, 4, ...
For Balmer: 2, 3, 4, 5, ...
For Paschen: 3, 4, 5, 6, ...

Wave Particle Duality - ooo00000!
Electron Gun Demo shows that electron behave the same way (i.e., they behave like waves).

**Electron Diffraction**

\[ 2d \sin \theta = n \lambda \]

LEADS TO

Laughe Matter Wave

\[ \lambda = \frac{h}{p} = \frac{h}{|mv|} = \frac{h}{m|v|} \]

A small particle has a big "Matter Wave" while a large particle has a small one.

**Bohr Model**

- Balance of electrostatic force = centrifugal force (as e⁻ rotates around nucleus)

\[ f_c = \frac{e^2}{4\pi\varepsilon_0 r^2} = f_c = \frac{me^2}{r} \]

Bohr assumed e⁻ can only stay on stable orbits (now e⁻'s cannot spiral into nucleus). Also, he said assumed angular momentum is quantized.
\[ \vec{r} \times \vec{p} = MeV \cdot c \times \frac{\hbar}{2\pi} = \hbar \]

Quantization of angular momentum implies a quantization of space.

RooyA!